# Vortex nucleation by Negative Ion in Superfluid $He^4$

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**Abstract**

In superfluid $He^4$, moving ions nucleate vortex loops by quantum tunneling through the barrier for $T < 0.3$ K, while thermal phonons activate the system over the barrier at higher temperature. The energy barrier height and the nucleation rate are calculated numerically. These are well fitted with the experimental data for low pressures $P < 16$ bar and low electric fields $E < 10^5$ V/m.

*Key words:* helium4;superfluidity;vortex;nucleation

## 1. Introduction

Some years ago, Hendry *et al* carried out detailed experiments of the vortex nucleation by negative ions [1]. Their results of the nucleation rate for low pressures ($\simeq 16$ bar) are well fitted by the equation

$$\nu(T, E) = \nu_0(E) + AT \exp(-\epsilon_b/k_b T). \qquad (1)$$

The first term of (1) describes the nucleation rate due to quantum tunneling through the potential barrier, while the second term of (1) involves thermal activation over the energy barrier $\epsilon_b$. Muirhead *et al* calculated the hydrodynamical energy of the vortex near the ion surface numerically, and according to their results, the energetically most favorable configuration of a nucleated vortex is a vortex loop on the equator of the ion [2]. This energy of the vortex loop is also calculated analytically by Ishikawa and Inoue using the approximation that an ion surface can be treated as the plane, and it gives similar results [3]. We regard Ishikawa-Inoue's energy as the potential of the vortex loop, and calculate the Scrödinger equation numerically. The energy barrier and the nucleation rate are derived from this calculation, and are compared with the experimental values.

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## 2. Calculation

To write the Shrödinger equation of the vortex loop, we should define the mass of the system. The length of a vortex loop is $(\pi + 2\tan^{-1}(r/R_I))r$ where $r$ is the radius of the vortex loop and $R_i$ is the radius of the ion. So the mass of the system is $m_c(\pi + 2\tan^{-1}(r/R_I))r$, where $m_c$ is the hydrodynamic mass of a vortex per unit length. However, if $r$ is smaller than the radius of the vortex core $a_0$, this mass is clearly not effective. Furthermore, if there is not enough space inside the vortex loop to go through a $He^4$ atom, that system cannot constitute a "vortex". We assume this limit is near $r = r_0 = \sigma/2 + a_0$, where $\sigma$ is the hard core length of a $He^4$ atom (see Fig. 1).

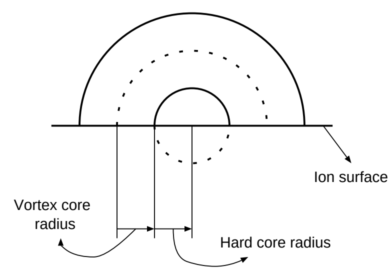


Fig. 1. The minimum state of the vortex loop.

We believe there is some kind of an excited state on the ion surface under this limit (such as a vortex of some shape with a surface oscillation), but figuring this out is a very difficult problem. So, we put the mass of the system $m = m_c(\pi + 2\tan^{-1}(r_0/R_I))r$ for all the values of $r$ under this limit. This procedure can remove the singularity of the mass when $r \to 0$.

Therefore, the Shrödinger equation of the vortex loop is

$$\left(-\frac{\hbar^2}{2f(r)}\frac{d^2}{dr^2} + V(r)\right)\psi(r) = \epsilon\psi(r), \qquad (2)$$

where $f(r)$ is

$$f(r) = \begin{cases} m_c(\pi + 2\tan^{-1}(r/R_I))r & (r \geq r_0) \\ m_c(\pi + 2\tan^{-1}(r_0/R_I))r_0 & (r < r_0). \end{cases} \qquad (3)$$

## 3. Results

We calculate the wavefunction $\psi(r)$ and the eigenvalue $\epsilon$ in (2) numerically. For the nucleation process, we set the critical velocity of the ion $U_c$ at the velocity just when the expectation value $\langle r \rangle$ takes larger value than the position of the potential maximum. Using the velocity just before this $U_c$, we regard the gap between the height of the potential maximum and the energy of the vortex at this velocity as the energy barrier. The results are shown in Fig. 2. These results are of the

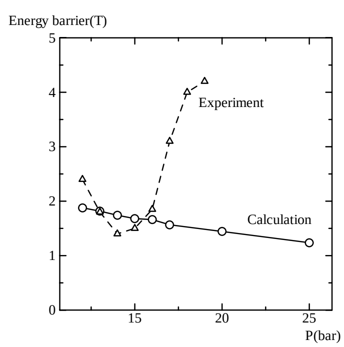


Fig. 2. The energy barrier plotted against pressure P.

same order as the experimental ones for the pressures under 16 bar. However, it is clear that there exists considerable disagreement for high pressures. This raise of the energy barrier at high pressures in the experimental results cannot explain by the vortex model we have adopted, as the earlier works showed [1] [2].

The nucleation rate is calculated by the WKB approximation, and taking the average over the ion-velocity $U_i$:

$$\nu(E) = \int_{U_c}^{U_l} R(U_i) f(U_i, E) P(U_i) dU_i. \qquad (4)$$

Here, $R(U_i)$ is the nucleation rate at $U_i$ which is calculated by WKB approximation. $P(U_i)$ is the rate that two-roton emission process does not occur before the system nucleate the vortex. $f(U_i, E)$ is the velocity-distribution function of the ion [4], and $U_l$ is the upper limit for the nucleation process.

Our results of the nucleation rates by quantum tunneling at $P = 13, 15$ bar are shown in Fig. 3.

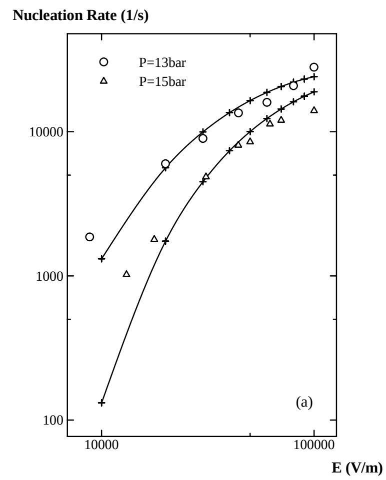


Fig. 3. The nucleation rates at $P = 13, 15$ bar. The data points have been taken from the experiment of Hendry *et al.* [1]

We focused the nucleation results for $E < 10^5$ V/m, because it is supposed that the increase of the nucleation rate for $E > 10^5$ V/m in the experiment is due to the other, second nucleation mechanism, which we do not know. In the range of the electric fields $E < 10^5$ V/m, and the pressures under 16 bar, our results are well fitted with the experiment. Nevertheless, at the pressures over 16 bar our results do not correspond with the experimental ones. It is possible that the model we have adopted cannot well describes the situation at these pressures, and this causes the disagreement of the energy barrier and the nucleation rate. These problems need another new mechanism of the nucleation, and more information about the vortex and the roton.

## References

[1] P.C. Hendry, N.S. Lawson, P.V.E. McClintock, C.D.H. Williams, R.M. Bowley, Phil. Trans. Roy. Soc. Lond. A **332** (1990) 387.

[2] C.M. Muirehad, W.F. Vinen, R.J. Donnelly, Phil. Trans. Roy. Soc. Lond. A **311** (1984) 433.

[3] K. Ishikawa, I. Inoue, Czech. J. Phys. **46** Suppl.S1. (1996) 371.

[4] R.M. Bowley, F.W. Sheard, Phys. Rev. B **16** (1977) 244.