Paramagnetism of Two-Dimensional Quantum Dots with Superconducting Core

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Abstract

We study orbital magnetism of a mesoscopic superconductor/normal metal hybrid system. We especially consider a two-dimensional normal metal quantum dot with superconducting core. The Bogoliubov-de Gennes equation is solved numerically and the energy levels of the quasiparticle is obtained. It is found that characteristic energy levels appear below the superconducting gap and make significant paramagnetic contributions to the magnetization. We also estimate these subgap levels analytically by using the WKB approximation and clarify their physical properties.

Key words: Orbital Magnetism; Mesoscopic Superconductivity; Quantum Dot; Proximity Effect

1. Introduction

For the last few decades, progress has been remarkable in mesoscopic superconducting systems and several intriguing phenomena have been found. Anomalies of magnetic responses have been reported in cylindrical samples with superconductor (S) / normal metal (N) junctions at very low temperatures [1]. In the experiments, the Meissner diamagnetism induced by the proximity effect is suppressed drastically and even a paramagnetic behavior is observed, which implies that there is a large paramagnetism comparable to the Meissner diamagnetism. Several theoretical studies have been devoted to the origin of the paramagnetism: "π-state" localized at the S/N interface [2] and "glancing state" localized at the outer edge of N [3]. However, these edge states have not explained the anomalies completely [1].

This problem is related to the orbital magnetism in S/N hybrid mesoscopic systems. Orbital magnetism has been well studied on quantum dots of N, and it is known that mesoscopic fluctuation is significant at low temperatures and it is very sensitive to the field dependence of the energy spectrum near the Fermi level (FL). For S/N hybrid systems, as the temperature is decreased below the critical temperature of S, the superconducting correlation (SC) affects the energy levels in the N region, particularly, near the FL, as is known as the "proximity effect". Hence, it is expected that the affected energy levels may result in a large paramagnetism. To clarify whether the paramagnetism appears actually for the S/N hybrid system and, if it exits, whether it is attributed to some edge states, in this paper we perform a calculation on a simplified two-dimensional model.

2. Magnetization and Energy Levels

We consider electrons of mass m and charge $-e$ confined in a quantum dot with a cylindrical hard confining potential in a magnetic field. The disk consists of S and N region, and the N region surrounds the S region (the inner radius $R_{\rm S}$, and the outer radius $R_{\rm N}$). We solve numerically the Bogoliubov-de Gennes equation, assuming that the pair potential is non-zero only in the S region, and the vector potential $\mathbf{A}$ yields the uniform magnetic field B outside the S region. By using the ob-

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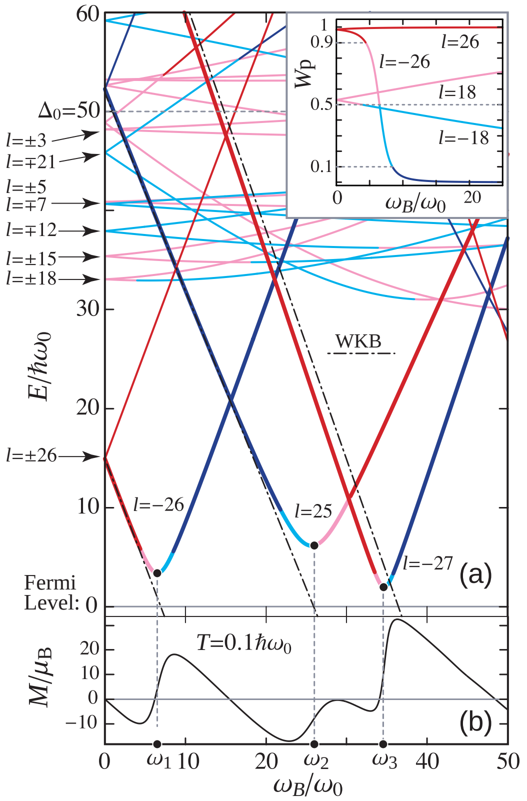

Fig. 1. Energy levels $E_j(B)$ and the magnetization curve $M(B)$.

tained energy levels E_j and wave functions (u_j, v_j) of the quasiparticle, we first calculate the current density distribution, $\mathbf{J} = \mathrm{Re}\langle\hat{\Psi}^\dagger(-e/m)\,(\mathbf{p}+e\mathbf{A}/c)\,\hat{\Psi}\rangle$, and then the magnetization M as the dipole moment of total electrons in the dot.

Figure 1(a) shows the field dependence of the energy levels $E_j(B)$ for $R_\mathrm{S}/R_\mathrm{N} = 0.8$, and fig. 1(b) shows the magnetization curve $M(B)$ at a low temperature ($T = 0.1\hbar\omega_0$), where we take the energy unit as $\hbar\omega_0 = \hbar^2/(2mR_\mathrm{N}^2)$. We set the chemical potential and the superconducting gap as $\mu = 1000\hbar\omega_0$ and $\Delta_0 = 50\hbar\omega_0$. We also take the unit of magnetization as the Bohr magneton μ_B, and the magnetic field B is shown by the cyclotron frequency ω_B. In fig. 1(a), we can actually see that several levels lie near the gap Δ_0 away from the FL and, furthermore, some levels approach and leave the FL, which cause the mesoscopic fluctuation of the magnetization as seen from fig. 1(b). The characteristic levels are specified by the angular momentum quantum number $l = -26, +25, -27$ in fig. 1(a). The low-lying states (LLS) have large field dependence and get closest to the FL at the field $\omega_i (i = 1, 2, 3)$, where the SC is the strongest, which is known by estimating the weight of the "particle" component, $W_\mathrm{p} = \int \mathrm{d}^2r|u_j|^2 = 1 - \int \mathrm{d}^2r|v_j|^2$, [see inset in fig. 1(a)]. Spontaneously, the slope of magnetization curve in fig. 1(b) takes the local maxima and the susceptibility shows paramagnetic peaks with suppressing a large diamagnetism in the background.

From our numerical findings, we know: (1) The LLS are almost unaffected by the SC in the entire field range excluding the neighborhood of the fields ω_i. (2) Just at ω_i, the LLS are influenced strongly by the SC, and the dominant components are switched. (3) The amplitudes of the wave functions of LLS are concentrated mainly in the N region. The properties (1)-(3) can be interpreted qualitatively in terms of the quasi-classical orbit, and confirmed quantitatively by using the WKB approximation. The LLS correspond to the "glancing state" [3] excluding the neighborhood of ω_i, where the Andreev bound state is established. Hereafter we set $\hbar = \omega_0 = R_\mathrm{N} = 1$. For the LLS in the weak field region involving $\omega_B = 0$, we obtain asymptotically

$$\mu \pm E_j \sim l^2\left(1 + y_l^2\right) + \frac{l}{2}\left[1 - R_\mathrm{S}^2\left(1 + \tfrac{2}{3}y_l^2\right)\right]\omega_B, \quad (1)$$

where $y_l \sim O\left(|l|^{-1/3}\right)$ as $\mu \to \infty$, since $|l| > R_\mathrm{S}\sqrt{\mu}$ holds. Equation (1) is plotted by the dash-dot line in fig. 1(a), which shows good agreement. We find that there is a maximum ω_i, which is estimated as

$$\max(\omega_i) \sim 2(9\pi/4)^{\frac{2}{3}}\left(1 - R_\mathrm{S}^2\right)^{-1}\mu^{\frac{1}{6}}. \quad (2)$$

For the sample with $R_\mathrm{S}/R_\mathrm{N} \approx 0.71$, $R_\mathrm{N} \approx 12\mu$m, and $\mu \approx 5.4$eV, eq. (2) yields $\max(\omega_i) \sim 19$Oe, and it coincides with the field at which the Meissner diamagnetism starts degrading in the experiment [1], although the actual relation between the experiment and our findings requires a further study. The detail will be published elsewhere.

Acknowledgements

This work was supported by a Grant-in-Aid for Scientific Research (C), No. 14540321 from MEXT. H. I. was supported by MEXT, Grant-in-Aid for Encouragement of Young Scientists, No. 13740197.

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