Conductance renormalization and conductivity in quatum wires with multiple subbands

Takashi Kimura [1]

Advanced Research Institute for Science and Engineering, Waseda University, 3-4-1 Ohkubo, Tokyo 169-8555, Japan

Abstract

Using a Tomonaga-Luttinger model, we studied the conductance renormalization and conductivity of a quantum wire with multiple subbands. As in a single-band system, the conductance of a quantum wire with an arbitrary number of subbands is not renormalized by the electron-electron interaction. For a dirty Tomonaga-Luttinger model with multiple subbands, we found that inter-subband interaction enhances the conductivity, which is contrary to that expected from the intra-subband repulsive interaction, and that the conductivity increases with increasing number of subbands.

Key words: conductance renormalization, conductivity, quantum wires, multisubband Tomonaga-Luttinger models

In one dimensional (1D) electron systems, the interaction between electrons has been of great importance not only in bulk systems but also in mesoscopic systems such as quantum wires. For example, transport properties of quantum wires have been extensively studied based on the Tomonaga-Luttinger (TL) liquid [2,3]. The conductance in a clean quantum wire with single band has been studied theoretically [4–7] and experimantally [8] and it is now confirmed that the conductance is not renormalized by the interaction between electrons.

Quantum wires with multiple subbands are also of interest. For example, a recent experiment [9] shows the conductance is smaller than the quantized conductance only in a high in-plane magnetic field, where the multiple inequivallent spin subbands cross the Fermi level. On the other hand, in a two-band model, which includes the backword scattering processes between electrons, Starykh et al. [10] have shown the conductance is not renormalized by the interaction between electrons.

[1] Present address: Department of Physics, University of Tokyo, 7-3-1 Hongo, Tokyo 113-0033, Japan, and Advanced Research Institute for Science and Engineering, Waseda University, 3-4-1 Ohkubo, Tokyo 169-8555, Japan. E-mail: kimura@kh.phys.waseda.ac.jp

In this paper, we study the conductance renormalization in a clean multi- (N-) subband TL model with the inter-subband forward scattering in order to clarify whether or not the conductance is renormalized within the multi-subband TL model. We assume the scattering processes with large momentum transfer such as backward scatterings can be neglected. We also study the conductivity in a dirty TL model with multiple subband since the interaction between electrons plays an important role in the temperature dependence of the conductivity [11–13].

The Hamiltonian of a clean N-subband spinless (or $N/2$ spinful) TL model is presented as

$$H = \sum_{i}^{N} \frac{1}{4\pi} \int dx \Big\{ v_N^i [\nabla\Theta_+^i(x)]^2 + v_J^i [\nabla\Theta_-^i(x)]^2 \Big\} + \sum_{i\neq j}^{N} \frac{1}{4\pi} \int dx \Big\{ \frac{g_N^{ij}}{2} [\nabla\Theta_+^i(x)][\nabla\Theta_+^j(x)] + \frac{g_J^{ij}}{2} [\nabla\Theta_-^i(x)][\nabla\Theta_-^j(x)] \Big\}. \quad (1)$$

Here i or j show the subband index. Θ_+^i is the phase variable for the i-th subband and Θ_-^i is its dual variable. $v_N^i \equiv v_F^i + g_4^i + g_2^i \equiv v^i/K^i$ and $v_J^i \equiv v_F^i + g_4^i - g_2^i \equiv v^i K^i$, where $g_{2(4)}^i$ is the interaction parameter between electrons with the opposite (same) velocity direction in the i-th subband. v_F^i is the Fermi velocity of the i-th subband and v^i (K^i) is the velocity of the excitation (critical exponent) of the i-th subband. The inter-subband forward scatterings are included through $g_N^{ij} \equiv g_4^{ij} + g_2^{ij}$ and $g_J^{ij} \equiv g_4^{ij} - g_2^{ij}$, where g_2^{ij} (g_4^{ij}) is the interaction parameter between electrons with the opposite (same) velocity direction in the i-th and j-th subbands.

Assuming $\hbar = e^2 = 1$, we obtain the dc mean current operator $\hat{j}_M^i \equiv \frac{1}{L}\int_0^L \ dx \hat{j}^i(x) = \frac{1}{L}\Big(v_J^i \hat{J}_i + \sum_{j(\neq i)} \frac{g_J^{ij}}{2}\hat{J}_j\Big)$. Here i and j are the subband indices. L is the system length. $\hat{j}^i(x)$ is the local current operator in the i-th subband. $\hat{J}_i \equiv \hat{N}_1^i - \hat{N}_2^i$ is the operator for the difference between total number of particles of right-going electrons ($\hat{N}_1^i$) and left-going ones ($\hat{N}_2^i$) [1]. Following Ref. [4] for a single-band system, the chemical potential difference between the right-going going electrons and the left-going electrons are obtained as $\delta\mu \equiv \mu_1 - \mu_2 = \frac{\partial E}{\partial N_1^i} - \frac{\partial E}{\partial N_2^i} = \frac{2\pi}{L} v_J^i J_i + \frac{\pi}{L}\sum_{j(\neq i)} g_J^{ij} J_j$ (for all i,j). The conductance is obtained by using $j_M^i \equiv (2v_J^i J_i + \sum_{j(\neq i)} g_J^{ij} J_j)/2L$ (the eigenvalue of $\hat{j}_M^i$) as $G = \sum_i j_M^i/\delta\mu = N/2\pi$. Hence, the conductance is not renormalized by the interaction between electrons. We stress that this result is independent of not only the strength of the interaction but also the number of subbands.

For the conductivity of a dirty quantum wire with multiple subbands, we assume a simplified TL model with N spinful subbands, in which the intra- or inter-subband spin-independent interactions and the Fermi velocities are independent of the subband ($g_2^i = g_4^i \equiv \pi v_F g/2$ and $g_2^{ij} = g_4^{ij} \equiv \pi v_F g'/2$ for all i,j). Following Ref. [14] for the Mori formalism [15], we find that the conductivity $\sigma(T)$ within the second order perturbation for the impurity scattering as

$$\sigma(T) \propto T^{2(1-K)}, \qquad (2)$$
$$K \equiv \frac{1}{N}\frac{1}{\sqrt{1+g+(N-1)g'}} + \Big(1-\frac{1}{N}\Big)\frac{1}{\sqrt{1+g-g'}}.$$

For $N=2$, the result of Ref. [13] is reproduced. Interestingly, $\frac{\partial K}{\partial |g'|} > 0$ always holds for arbitrary subband numbers. This means that the inter-subband interaction, being independent of its sign, enhances the conductivity. This is not trivial but may be understood as follows: The conductivity of the TL models is suppressed by the CDW correlation due to the impurity pinning. The inter-subband interaction disturbs the CDW correlation of each subband because of the discordance between the Fermi wave numbers of different subbands, resulting in an enhanced conductivity. On the other hand, $\frac{\partial K}{\partial N} > 0$ also always holds, and thus the conductivity is monotonically enhanced as a function of the number of subbands. This is because the inter-subband interaction, which enhances the conductivity, works more significantly for a larger number of subbands. If one compares this result with that of ref. [16] based on the Fermi liquid ($\sigma(T) \propto T^{1/N}$), there is a qualitative consistency in the sense that the conductivity is an increasing function of N.

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