Magnetic ground-state phase diagram of a multiple-spin exchange model on the triangular lattice

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Abstract

We investigated the ground state of a multiple-spin exchange model, which may describe magnetism of the solid ^{3}He layers adsorbed on graphite. The model contains four-spin exchange terms together with the conventional Heisenberg ones. Using the mean-field theory we found numerous ground-state phases with large number (up to 144) of sublattices. These phases replace the previously reported infinitely degenerate phase. A novel phase with 6-sublattice non-coplanar spin structure, where both vector and scalar chiral orders exist, appears between the 120° Néel and tetrahedral phases. This phase is replaced by a novel 12-sublattice phase under the magnetic field.

Key words: multiple-spin exchange; frustration; solid ^{3}He layer; phase diagram

Experimental studies on the magnetic and thermal properties of the solid ^{3}He layers adsorbed on graphite revealed that the system cannot be described by conventional Heisenberg model[1]. A peculiar double-peak structure was observed in the specific heat curve[2]. Multiple-spin exchange interactions are believed to be a clue to understand the magnetism in this system and several theoretical studies of the multiple-spin exchange model on the triangular lattice have been accomplished [3–6]. According to a mean-field theory assuming up to four sublattices, four ground state phases were found in the absence of the external field [4]. Succeedingly the so-called intermediate phase is reported to be infinitely degenerate[5]. Recently it is argued that there exist two kinds of spin-liquid phases[6,7]. The properties of the ground state, however, are still not well understood even at the level of the mean-field theory because of the strong frustration.

We investigate in this paper a model with four-spin exchanges together with conventional two-spin ones and determine the ground state in and without the magnetic field by a mean-field approximation. The model is described as

$$H = J \sum_{<i,j>}^{\mathrm{n.n.}} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + K \sum_p h_p - B \sum_i \sigma_i^z, \qquad (1)$$

where $\boldsymbol{\sigma}_i/2$ is the spin at the site i on the triangular lattice and $\sum_p$ denotes the sum over all possible diamonds composed of two unit triangles. B denotes the normalized external magnetic field applied along the z axis. The four-spin exchange term of the diamond p composed of spins 1, 2, 3 and 4 reads

$$h_{\mathrm{p}} = \sum_{1 \le i < j \le 4} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_4) + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_4)(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3)(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_4). \qquad (2)$$

In the mean-field theory we treat spins as classical vectors of length 1/2 and minimize the energy in terms of their orientations. We take K as the unit of the energy throughout this paper. We assume larger number of sublattices than 4, i.e., examine 6, 12, 18, 24, 36, 48, 72 and 144 sublattice structures, and search for the lowest energy state numerically using the conjugate gradient method.

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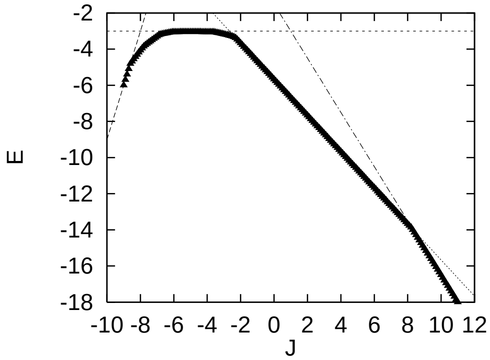

Fig. 1. Ground-state energy as a function of J.

We show the ground-state energy in the case of $B = 0$ in Fig. 1 as a function of J. The energy vs J curve is well approximated by four straight lines, which correspond to the energies of the ferromagnetic, intermediate (uuud), tetrahedral and 120° Néel phases, respectively. In particular for $-8 \lesssim J \lesssim -3$ the energy hardly depends on J and is slightly lower than 3, the energy of the previously reported infinitely degenerate phase. This result clearly shows that in this parameter range the variation in J does not contribute the energy gain due to strong frustration. In this region the ground state has a magnetic order with 144 sublattices. We found twelve ground-state phases in total for $B = 0$ (see Fig. 2).

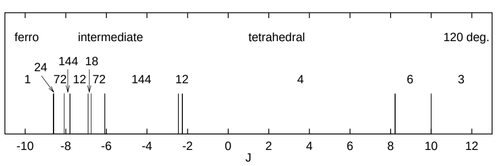

Fig. 2. Ground-state phase diagram for $B = 0$. Phase boundaries are shown by vertical bars. The numbers are of sublattices exhibited. The range of a 24-sublattice phase is very small ($\Delta J \simeq 0.02$).

Most of them are coplanar magnetic structures except for three phases, i.e. a phase with 144 sublattices, the previously reported tetrahedral phase and one with a six sublattices which appear for $-8.08 < J < -7.79$, $-2.26 < J < -8.22$ and $8.22 < J < 10$, respectively. Among them the six sublattice phase has been expected from the spin-wave theory in the 120 ° Néel phase[8]. The ground state in this phase has both antiferromagnetic scalar chiral order and ferromagnetic vector chiral order. The energy difference between this phase and the tetrahedral or the 120° Néel phase is quite tiny.

We also examined the ground state under the external magnetic field and found several new magnetic phases for $-8 \lesssim J \lesssim -7$ with 12, 48, 72 and 144 sublattices. Also a phase with 12 sublattices appears in a region where $4 \lesssim J \lesssim 11$ and $3 \lesssim B \lesssim 20$ as is shown in Fig. 3. This phase has a non-coplanar magnetic order without vector chiral order.

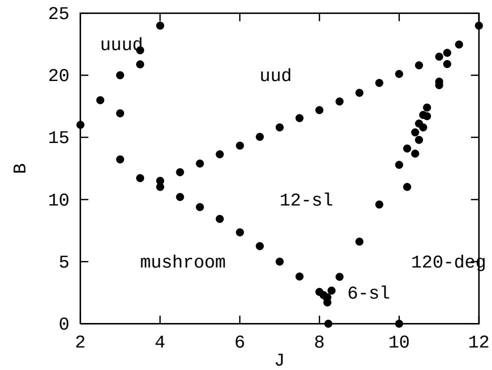

Fig. 3. Magnetic phase diagram for $2 < J < 12$. "12-sl" and "6-sl" denote phases with 12 and 6 sublattices, respectively.

Since we searched for structures with only up to 144 sublattices, we may not conclude all the phases found in this study to be true mean-field ground states. In particular the phases with 144 sublattices found in the intermediate region may be replaced with phases with larger spatial structures in further studies. Another possibility is a ground-state phase with continuously varying periodicity. We, however, believe the phases with 6 and 12 sublattices found for positive J are true classical ground states. A numerical diagonalization study reported that there may be a new type of spin liquid phase in the corresponding parameter region[7]. The relation between this spin liquid phase and the 6 sublattice phase found in the present study is quite intriguing. Details of the obtained phases will be reported elsewhere.

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