Symmetry classes of triplet vortex lattice solutions of the Bogoliubov de-Gennes equation in a square lattice

Akira Goto [a,1], Yoshiki Hori [a], Masa-aki Ozaki [b]

[a] *Kochi National College of Technology, Nankoku 783-8508, Japan*

[b] *Kumakouji 19-21 Uji City , Kyoto 611-0002, Japan*

Abstract

We give a group theoretical classification of the triplet vortex lattice states of the two-dimensional Hubbard model with a nearest neighbor ferromagnetic exchange interaction in a uniform magnetic field. We obtain 11 types of tetragonal vortex lattice states for the magnetic flux $\phi = \phi_0/p^2$ ($\phi_0 = ch/2e$ is the flux quantum, p is an integer) through a unit cell of crystal lattice. We show the configurations of the order parameters corresponding to axial phase, up spin phase, planar phase and bipolar phase. It is clarified what types of vortex lattice phase are possible in triplet superconductors such as Sr_2RuO_4 with basal square lattice when the symmetry of magnetic translation is considered.

Key words: triplet superconductivity; vortex lattice; symmetry; Sr_2RuO_4

Recently much attention has been focused on a spin triplet superconductivity of Sr_2RuO_4. Vortex lattice states for Sr_2RuO_4 have been studied on Ginzburg-Landau theory by Agterberg [1]. In our previous papers [2,3], we gave a classification of tetragonal vortex lattice solutions of singlet superconductivity in a two dimensional square lattice. Then it is interesting to know what types of triplet vortex lattice states are there in a two dimensional square lattice.

We consider the two dimensional Hubbard Hamiltonian with a nearest neighbor ferromagnetic exchange interaction ($J < 0$) in a magnetic field with a vector potential $\boldsymbol{A(r)}= (-\frac{1}{2}By, \frac{1}{2}Bx, 0)$:

$$\mathcal{H} = -t \sum_{(m,n)s} \{ \mathrm{e}^{\mathrm{i}Kn} a^\dagger_{(m,n)s} a_{(m+1,n)s} + \mathrm{e}^{-\mathrm{i}Km} a^\dagger_{(m,n)s} a_{(m,n+1)s} + \mathrm{h.c.}\} - \mu \sum_{(m,n)s} a^\dagger_{(m,n)s} a_{(m,n)s}$$
$$+ J \sum_{(m,n)} \sum_{s_1,s_2,s_3,s_4} \sum_{\lambda} \{ (a^\dagger_{(m,n)s_1} \sigma^\lambda_{s_1,s_2} a_{(m,n)s_2}) (a^\dagger_{(m+1,n)s_3} \sigma^\lambda_{s_3,s_4} a_{(m+1,n)s_4}) + (a^\dagger_{(m,n)s_1} \sigma^\lambda_{s_1,s_2} a_{(m,n)s_2}) (a^\dagger_{(m,n+1)s_3} \sigma^\lambda_{s_3,s_4} a_{(m,n+1)s_4})\}, \quad (1)$$

where μ is the chemical potential, (m,n) denotes a site in a square lattice, $K = \frac{\pi}{2}\frac{\phi}{\phi_0}, \phi = Ba^2, a$ is a lattice constant, $\phi_0 = \frac{ch}{2e}$ is the flux quantum. The symmetry group of the Hamiltonian is given by [2]

$$G_0 = (e + tC_{2x})\boldsymbol{C}_4\boldsymbol{T}\boldsymbol{S}\boldsymbol{\Phi}, \quad (2)$$

where t is the time reversal, $\boldsymbol{T}$ is the group of the magnetic translation [2,3,5] consisting of the elements $T(Ma\boldsymbol{e}_x + Na\boldsymbol{e}_y)(M, N =$integer) such that $T(Ma\boldsymbol{e}_x+Na\boldsymbol{e}_y)a^\dagger_{(m,n)s} = \mathrm{e}^{\mathrm{i}K(Mn-Nm)}\, a^\dagger_{(m+M,n+N)s}$, $\boldsymbol{C}_4$ is the four-fold rotation group, $\boldsymbol{S}$ is the group of the spin rotation ($SU(2)$), and $\boldsymbol{\Phi}$ is the group of the global gauge transformation.

Hereafter we restrict our consideration to the case $\phi = \phi_0/p^2$ (p =integer) for an illustrative purpose. Then we can define an invariance magnetic translation group $\boldsymbol{L}$ for a tetragonal vortex lattice [2]. $\boldsymbol{L}$ is a subgroup of $\boldsymbol{T\Phi}$ consisting of elements $L(Mpa\boldsymbol{e}_x+Npa\boldsymbol{e}_y)$ such that

[1] E-mail: goto@ge.kochi-ct.ac.jp

Table 1
Invariance groups of triplet vortex lattices

		Invariance group
axial phase	$G^z_{(0)}$	$= (1+tC_{2x})(e+u_{2x}\tilde{\pi})\boldsymbol{C}_4^{(0)}A(\boldsymbol{e}_z)\boldsymbol{L}$
	$G^z_{(2)}$	$= (1+tC_{2x})(e+u_{2x}\tilde{\pi})\boldsymbol{C}_4^{(2)}A(\boldsymbol{e}_z)\boldsymbol{L}$
	$G^z_{(+1)}$	$= (1+tC_{2x})(e+u_{2x}\tilde{\pi})\boldsymbol{C}_4^{(+1)}A(\boldsymbol{e}_z)\boldsymbol{L}$
	$G^z_{(-1)}$	$= (1+tC_{2x})(e+u_{2x}\tilde{\pi})\boldsymbol{C}_4^{(-1)}A(\boldsymbol{e}_z)\boldsymbol{L}$
up spin phase	$G^{\tilde{z}}_{(0)}$	$= (1+tC_{2x}u_{2x})\boldsymbol{C}_4^{(0)}\tilde{A}(\boldsymbol{e}_z)\boldsymbol{L}$
	$G^{\tilde{z}}_{(2)}$	$= (1+tC_{2x}u_{2x})\boldsymbol{C}_4^{(2)}\tilde{A}(\boldsymbol{e}_z)\boldsymbol{L}$
	$G^{\tilde{z}}_{(+1)}$	$= (1+tC_{2x}u_{2x})\boldsymbol{C}_4^{(+1)}\tilde{A}(\boldsymbol{e}_z)\boldsymbol{L}$
	$G^{\tilde{z}}_{(-1)}$	$= (1+tC_{2x}u_{2x})\boldsymbol{C}_4^{(-1)}\tilde{A}(\boldsymbol{e}_z)\boldsymbol{L}$
planar phase	G_{II}	$= (1+tC_{2x}u_{2x})(e+\tilde{\pi}u_{2z})_{II}\boldsymbol{C}_4\boldsymbol{L}$
bipolar phase	G^+_{sII}	$= (1+tC_{2x})(e+\tilde{\pi}u_{2z})_{II}\widetilde{\boldsymbol{C}}_4^+\boldsymbol{L}$
	G^-_{sII}	$= (1+tC_{2x})(e+\tilde{\pi}u_{2z})_{II}\widetilde{\boldsymbol{C}}_4^-\boldsymbol{L}$

$\boldsymbol{C}_4^{(0)} = \{e, C^+_{4z}, C_{2z}, C^-_{4z}\}$, $\boldsymbol{C}_4^{(2)} = \{e, \tilde{\pi}C^+_{4z}, C_{2z}, \tilde{\pi}C^-_{4z}\}$,
$\boldsymbol{C}_4^{(+1)} = \{e, \widetilde{(\pi/2)}C^+_{4z}, \tilde{\pi}C_{2z}, \widetilde{(-\pi/2)}C^-_{4z}\}$,
$\boldsymbol{C}_4^{(-1)} = \{e, \widetilde{(-\pi/2)}C^+_{4z}, \tilde{\pi}C_{2z}, \widetilde{(\pi/2)}C^-_{4z}\}$,
$A(\boldsymbol{e}_z) = \{u(\boldsymbol{e}_z,\theta)|0 \le \theta \le 2\pi\}$, $\tilde{A}(\boldsymbol{e}_z) = \{u(\boldsymbol{e}_z,\theta)\tilde{\theta}|0 \le \theta \le 2\pi\}$,
${}_{II}\boldsymbol{C}_4 = \{e, C^+_{4z}u(\boldsymbol{e}_z,\pi/2), C_{2z}u(\boldsymbol{e}_z,\pi), C^-_{4z}u(\boldsymbol{e}_z,-\pi/2)\}$,
${}_{II}\tilde{\boldsymbol{C}}_4^{\pm} = \{e, \widetilde{(\pi/2)}^{\pm}C^+_{4z}u_{2a}, \tilde{\pi}C_{2z}, \widetilde{(\pi/2)}^{\mp}C^-_{4z}u_{2a}\}$

$$L(\,M\,pa\boldsymbol{e}_x + Npa\boldsymbol{e}_y)\cdot a^\dagger_{(m,n)s} \equiv \mathrm{e}^{\mathrm{i}\frac{\pi}{2}(MN+M+N)}T(Mpa\boldsymbol{e}_x + Npa\boldsymbol{e}_y)\cdot a^\dagger_{(m,n)s} \quad (3)$$

Using a similar method with our previous paper [2,4], we obtain 11 types of triplet tetragonal vortex lattice states. Their invariance groups are listed in Table 1. Local bond order parameters(LBOD) are defined by $\langle a_{(m,n)s}a_{(m+1,n)s'}\rangle$ and $\langle a_{(m,n)s}a_{(m,n+1)s'}\rangle$. Those are shown in Fig. 1. Both axial and up spin phases, showing the same vortex lattice pattern depicted in (a), have 4 types of symmetries corresponding to $\boldsymbol{C}_4^{(+1)}(l=1)$, $\boldsymbol{C}_4^{(-1)}(l=-1)$, $\boldsymbol{C}_4^{(0)}(l=0)$ and $\boldsymbol{C}_4^{(2)}(l=2)$. Those are essentially one complex component order parameters and we show their phase parts only. For one planar phase and two bipolar phases("+" and "−"), each has two components("up" and "down") and those phase patterns are depicted in (b) and (c).

In order to investigate their topological properties, we have to introduce triplet local symmetric order parameters at each site (m,n). These are similar to those defined in the previous paper [3], so we omit their definitions. Each LBOP are decomposed to S, D, P wave components and each shows the definite winding numbers. These results are to be published elsewhere.

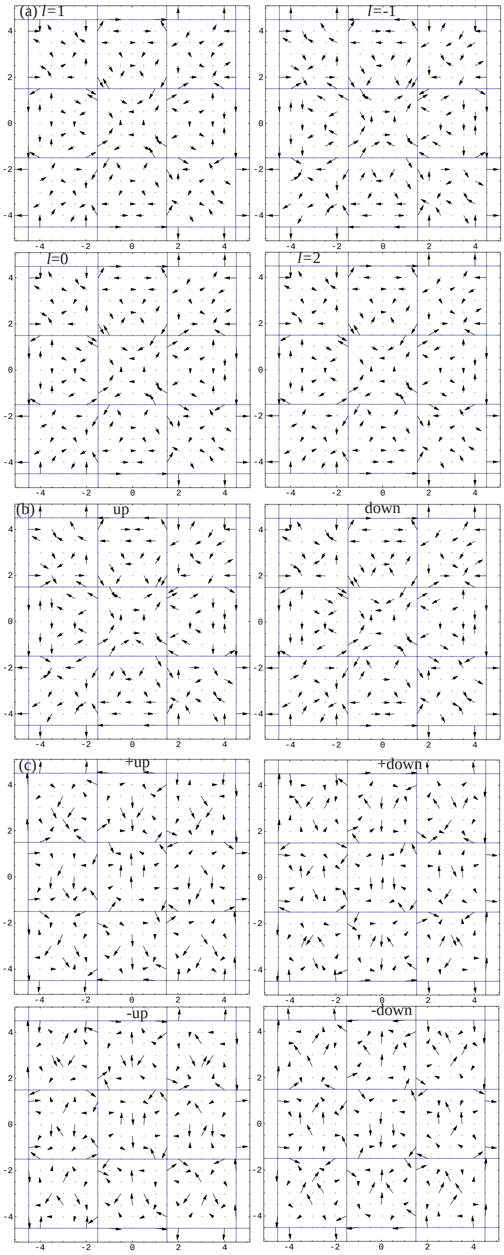

Fig. 1. Symmetries of the LBOPs for (a) axial(up spin) phase $G^z_{(l)}(G^{\tilde{z}}_{(l)})(l = 0, \pm 1, 2)$; $\langle a_{(m,n)\downarrow}a_{(m+1,n)\uparrow}\rangle$, $\langle a_{(m,n)\downarrow}a_{(m,n+1)\uparrow}\rangle$ ($\langle a_{(m,n)\uparrow}a_{(m+1,n)\uparrow}\rangle$, $\langle a_{(m,n)\uparrow}a_{(m,n+1)\uparrow}\rangle$), (b) planar phase G_{II}; $\langle a_{(m,n)s}a_{(m+1,n)s}\rangle$, $\langle a_{(m,n)s}a_{(m,n+1)s}\rangle$ ($s = \uparrow$(left), $\downarrow$(right)), (c) bipolar phase G^+_{sII}(upper), G^-_{sII}(lower); $\langle a_{(m,n)s}a_{(m+1,n)s}\rangle$, $\langle a_{(m,n)s}a_{(m,n+1)s}\rangle$ ($s = \uparrow$(left), $\downarrow$(right)). $p = 3$. 3×3 vortex lattice patterns are shown.

References

[1] D. F. Agterberg, Phys. Rev. B **58** (1998) 14484.

[2] M. Ozaki, M. Yamazaki, A. Goto and Y. Hori, Prog. Theor. Phys. **100** (1998) 253.

[3] M. Ozaki, Y. Hori and A. Goto, Prog. Theor. Phys. **101** (1999) 769.

[4] M. Ozaki, K. Machida and T. Ohmi, Prog. Theor. Phys. **74** (1985) 221; **75** (1986) 442.

[5] E. Brown, Phys. Rev. **133** (1964) A1038.