Quasi-particles in the mixed state of d-wave superconductors

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Abstract

The extended quasi-particle states in the mixed state of d-wave superconductors are investigated on the basis of the BdG equation. It is shown that the quasi-particle eigen states can be classified in terms of new topological quantum numbers in the presence of vortices. A new approximate scheme for solving the BdG equation is developed in the region $H_{c1} \ll H \ll H_{c2}$.

Key words: BdG equation ; mixed state ;d-wave superconductors ; quantum oscillations

To establish the correct description for the extended quasi-particles in the mixed state of d-wave superconductors is one of the current subjects in high-T_c superconductors. In this paper we give a new insight into the quasi-particle eigen states in the presence of vortices and present solutions of the BdG equation in the field region $H_{c1} \ll H \ll H_{c2}$ in d-wave superconductors.

Consider the BdG equation for a d_{xy}-wave superconductors in the following form,

$$\begin{bmatrix} h_0(\nabla) & \hat{\Delta}(\nabla) \\ \hat{\Delta}^*(\nabla) & -h_0^*(\nabla) \end{bmatrix} \begin{bmatrix} u_\alpha(\mathbf{r}) \\ v_\alpha(\mathbf{r}) \end{bmatrix} = E \begin{bmatrix} u_\alpha(\mathbf{r}) \\ v_\alpha(\mathbf{r}) \end{bmatrix}, \qquad (1)$$

where $h_0(\nabla) = \frac{1}{2m}\big(\mathbf{p} - \frac{e}{c}\mathbf{A}(\mathbf{r})\big)^2 - \varepsilon_F$ and $\hat{\Delta}(\nabla)$ is the gap operator defined as $\hat{\Delta} = -\frac{1}{k_F^2}\big\{\partial_x, \big\{\partial_y, \Delta(\mathbf{r})\big\}\big\} - \frac{\mathrm{i}}{4}\Delta(\mathbf{r})\big\{\partial_x, \partial_y\big\}\phi$ with $\{A, B\} \equiv \frac{1}{2}(AB + BA)$ [1,2]. In Eq.(1) the gap parameter is expressed as $\Delta(\mathbf{r}) = |\Delta(\mathbf{r})|\mathrm{e}^{i\phi(\mathbf{r})}$, $\phi(\mathbf{r})$ being the phase. For the extended quasi-particle states we can utilize the approximation $|\Delta(\mathbf{r})| \simeq \Delta_0$ in the region $H_{c1} \ll H \ll H_{c2}$. In the N-vortex state $\phi(\mathbf{r})$ is a multi-valued function having the topological singularity, $\nabla \times \nabla\phi(\mathbf{r}) = 2\pi \sum_i^N \delta(\mathbf{r}\text{-}\mathbf{R}_i)\mathbf{e}_z$, where $\mathbf{R}_i$ is the position of ith vortex. Anderson proposed a transformation for the wave-functions as $u_\alpha \to u_\alpha$, $v_\alpha \to v_\alpha \mathrm{e}^{-i\phi}$, or $u_\alpha \to u_\alpha \mathrm{e}^{i\phi}$, $v_\alpha \to v_\alpha$, to express the BdG equation in terms of only single-valued functions, eliminating the phase factor in Eq.(1)[3]. As pointed out by Franz and Tešanović, the phase factor can be eliminated generally by the transformation, $u_\alpha \to \mathrm{e}^{\mathrm{i}\phi_e} u_\alpha, v_\alpha \to \mathrm{e}^{-\mathrm{i}\phi_h} v_\alpha$, if $\phi_e + \phi_h = \phi$ [4]. The phase factor in Eq.(1) can really be eliminated by these transformations, but the essentials lying behind them are in the topological nature of the quasi-particle wave-functions, that is, the quasi-particle wave-function in the mixed state is a one in a multi-connected system. Therefore, $u_\alpha(\mathbf{r})$ and $v_\alpha(\mathbf{r})$ are path-dependent single-valued functions in the mixed state. On the basis of these observations we can prove that the wave-functions are expressed as

$$\begin{cases} u_\alpha(\mathbf{r}) = \tilde{u}_{n\mu}(\mathbf{r})\mathrm{e}^{\mathrm{i}(\frac{1}{2}-\mu)\phi(\mathbf{r};\mathbf{R_1},\cdots,\mathbf{R_N})} \\ v_\alpha(\mathbf{r}) = \tilde{v}_{n\mu}(\mathbf{r})\mathrm{e}^{-\mathrm{i}(\frac{1}{2}+\mu)\phi(\mathbf{r};\mathbf{R_1},\cdots,\mathbf{R_N})} \end{cases}, \qquad (2)$$

when the phase factor is explicitly extracted. In Eq.(2) μ is a *half and integer*, namely, $\mu = \pm\frac{1}{2}, \pm\frac{3}{2}, \cdots$, which may be interpreted as a *quantum number specifying the quasi-particle eigen-states* in the N-vortex state, and n denotes symbolically the other quantum numbers. The proof will be given in a separate paper. Then. substituting Eq.(2) into Eq.(1), we obtain the equation in terms of only single-valued functions as

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$$\begin{bmatrix} \hat{h}_0(\nabla;\mu) & \Pi(\nabla;\mu) \\ \Pi^*(\nabla;-\mu) & -\hat{h}_0^*(\nabla;-\mu) \end{bmatrix} \begin{bmatrix} \tilde{u}_{n\mu} \\ \tilde{v}_{n\mu} \end{bmatrix} = E \begin{bmatrix} \tilde{u}_{n\mu} \\ \tilde{v}_{n\mu} \end{bmatrix}, \quad (3)$$

where $\hat{h}_0(\nabla;\mu) = -\frac{\hbar^2}{2m}\left[\nabla + \mathrm{i}\frac{m}{\hbar}\mathbf{v}_s - \mathrm{i}\mu\nabla\phi\right]^2 - \varepsilon_F$, $\mathbf{v}_s$ is the superfluid velocity defined by $\mathbf{v}_s = \frac{\hbar}{m}\left(\nabla\phi - \frac{2e}{\hbar c}\mathbf{A}\right)$, and the off-diagonal component, $\Pi(\nabla;\mu)$, is given as

$$\Pi(\nabla;\mu) = -\frac{\Delta_0}{k_F^2}\Big(-\mu^2\partial_x\phi\cdot\partial_y\phi - \mathrm{i}\mu\{\partial_x,\partial_y\}\phi - \mathrm{i}\mu\big(\partial_y\phi\cdot\partial_x + \partial_x\phi\cdot\partial_y\big) + \partial_x\partial_y\Big). \quad (4)$$

Let us now solve Eq.(3) in the region $H_{c1} \ll H \ll H_{c2}$. In this field region the spatial variation of the magnetic field $\mathbf{B(r)}$ can be neglected in the 0th order approximation. Then, the vector potential in Eq.(3) is approximated as $\mathbf{A(r)} \simeq \mathbf{A}_0(\mathbf{r}) = (0, Bx, 0)$ with $B = \langle B_z(\mathbf{r})\rangle_{av}$. Furthermore, in this approximation we can assume $\mathbf{v}_s \simeq 0$, noting the relation, $\nabla\times\mathbf{B} = \frac{4\pi e^*}{c}\rho_s(\mathbf{r})\mathbf{v}_s(\mathbf{r}) \simeq 0$, where $\rho_s(\mathbf{r})$ is the local superfluid density and is finite outside the vortex cores. Then, we can utilize the approximate relation, $\nabla\phi(\mathbf{r}) \simeq \frac{2e}{\hbar c}\mathbf{A}_0(\mathbf{r})$. Note that the topological singularity in this approximation is reduced to $(\nabla\times\nabla\phi)_z = (\partial_x\partial_y - \partial_y\partial_x)\phi = 2\pi m$, with m being an integer. This result is equivalent to that in the "continuity approximation", i.e., $(\nabla\times\nabla\phi)_z \simeq 2\pi\langle\sum_i^N \delta(\mathbf{r}\text{-}\mathbf{R}_i)\rangle_{av} = 2\pi m$. Under these approximations the BdG equation is expressed as

$$\begin{bmatrix} \hat{h}_0(P,Q) & \Pi(P,Q;\mu) \\ \Pi^*(P,Q;-\mu) & -\hat{h}_0(P,Q) \end{bmatrix} \begin{bmatrix} \tilde{u}_{n\mu} \\ \tilde{v}_{n\mu} \end{bmatrix} = E \begin{bmatrix} \tilde{u}_{n\mu} \\ \tilde{v}_{n\mu} \end{bmatrix}, \quad (5)$$

where $\hat{h}_0(P,Q) = \mu\omega_c\big(Q^2+P^2\big) - \varepsilon_F$ and $\Pi(P,Q;\mu) = \frac{\Delta_0}{2\varepsilon_F}\hbar\omega_c\big[\mathrm{i}\mu + 2\frac{\mu}{\hbar}PQ\big]$. Here, $\omega_c = \frac{eB}{mc}$ is the cyclotron frequency and $Q = \left(\mu\frac{2e}{c}B\right)^{-\frac{1}{2}}p_x$, $P = \left(\mu\frac{2e}{c}B\right)^{-\frac{1}{2}}\left(p_y - \mu\frac{2e}{c}Bx\right)$ with $p_\alpha = -\mathrm{i}\hbar\partial_\alpha$, which satisfy the canonical commutation relation, $[Q,P] = \mathrm{i}\hbar$. Eq.(5) can be solved by means of the Landau level expansion for u_α and v_α. As seen in Eq.(5), the eigenstates can be specified by two quantum numbers, n and μ. The quantum number n is related to the Landau-level index, which is not a good quantum number in the superconducting state since the off-diagonal components in Eq.(5) induce the mixing of the levels. On the other hand, μ may be understood to be a topological one, which relates to the homotopy class of the classical orbits of the quasi-particles.

Let us now present numerical results for the energy eigen-values of Eq.(5). In Fig.1 we plot the field dependence of the lowest excitation energy in the case of $\varepsilon_F = 200$ meV, $\Delta_0 = 2$ meV and $m = m_e$ (free electron mass). In this choice of the parameter values we find $\hbar\omega_c = \Delta_0$ for $B \sim 16T$. Then, the crossover behavior of the excitation energy is expected to be seen in

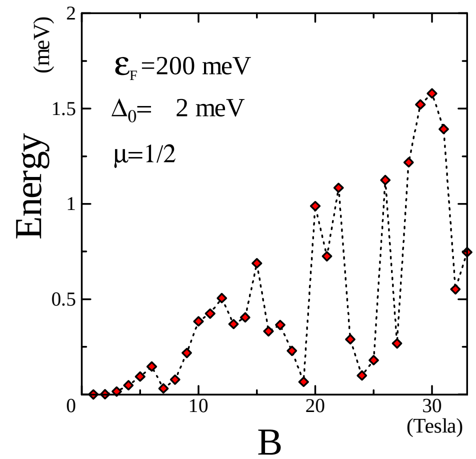

Fig. 1. The field dependence of the lowest excitation energy.

the field dependence given in Fig.1. First we note that the lowest energy eigen-state has zero-energy in the weak field region, but it abruptly changes to a gapped state above some value of B, which indicates that the quasi-particle eigen-state acquire a full energy-gap in the strong field region, though the symmetry of the gap function is the same as in the Meissner state. In the strong field region the quasi-particles traveling in the nodal directions can change directions into those with an energy-gap by the effect of the Lorentz force. This is the origin of the gapped superconducting state in the strong field region. Furthermore, we notice in Fig.1 that the lowest excitation energy shows oscillatory behavior in the field region of $\hbar\omega_c > \Delta_0$. These oscillations reflect the field dependence of the diagonal components of Eq. (5), that is, they are the quantum oscillations which leads to the de Haas van Alphen effect.

In summary our theory for the quasi-particles in d-wave superconductors can describe the crossover behavior from the gappless phase in the weak field region to the gapped phase showing the quantum oscillations in the strong field region.

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