Vortex loop fluctuations in Casimir thinning of helium films

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Abstract

A vortex-loop renormalization theory is used to calculate the Casimir forces that lead to the thinning of ^{4}He films near the superfluid transition point. For periodic boundary conditions the force is appreciable when the loop size becomes comparable to the film thickness. The results for T < T$_c$ match well with a 2-loop ϵ-expansion by Dietrich and Krech valid for T > T$_c$.

Key words: vortex loops; Casimir effect ; helium films ;

Critical fluctuations in a finite-size superfluid lead to a free-energy difference between the finite-size system and the bulk. If there is a connection between the two, such as between a saturated helium film and a bulk liquid reservoir, forces (called Casimir forces, in analogy with electromagnetic fluctuations) will develop that lead to a thinning of the film in the vicinity of the superfluid transition, an effect which has been observed experimentally [1,2]. Existing theories of the effect are incomplete: perturbative ϵ-expansion theories [3] are only able to handle the region T > T$_c$, and are able to describe only a tiny fraction of the experimental dip in the film thickness [2]. The superfluid regime T < T$_c$ is apparently far more difficult for the perturbation theories, and such calculations have not yet been attempted.

We show here that vortex loop excitations [4,5] are the critical fluctuations giving rise to the film thinning, and that vortex renormalization techniques provide a very simple means of calculating the Casimir force in the regime T < T$_c$.

The difference in free energy per unit area between the film of thickness L and the bulk is given by $\delta F = L(f_f(L) - f_b)$ where f_b and f_f are the free energies per unit volume of the bulk and film. In the vortex-loop renormalization scheme these free energies [5] can be written as a sum over the average loop diameter a,

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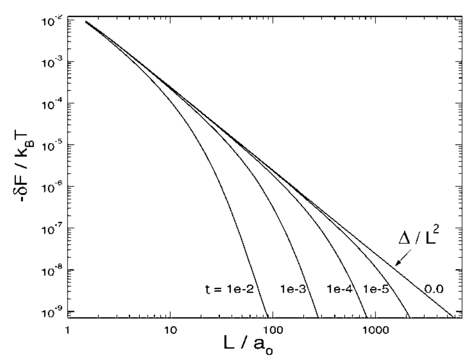

Fig. 1. Free-energy difference between bulk and film as a function of the film thickness L, for several reduced temperatures $t = (T_c - T)/T_c$.

$$\frac{\delta F}{k_B T} = -L \frac{\pi}{a_o^3} \int_{\beta L}^{\infty} \left(\frac{a}{a_o}\right)^{-2} \exp\left(-U(a)/k_B T\right) \frac{da}{a_o} \qquad (1)$$

where a_o is the bare core diameter, U(a) the renormalized loop energy [4], and βL is the maximum loop size in the film; a comparison [6] with a Monte Carlo simulation gave $\beta = 0.75$. In Ref. [4] it was noted that the Boltzmann factor in Eq. 1 can be written in the form

$$\exp\left(-U(a)/k_B T\right) = \frac{3\, a_o}{4\pi^3} \left(\frac{a}{a_o}\right)^{-6} \frac{\partial}{\partial a} \left(\frac{1}{K_r}\right) \qquad (2)$$

where $K_r = \hbar^2 \rho_s a_o / m^2 k_B T$ is the dimensionless su-

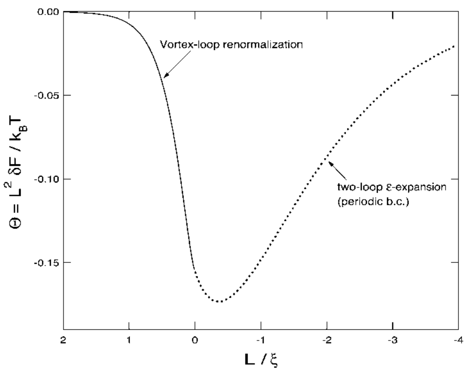

Fig. 2. Free energy scaling function versus L/ξ, where ξ is the bulk correlation length, taken to be positive for T $<$ T$_c$ and negative for T $>$ T$_c$.

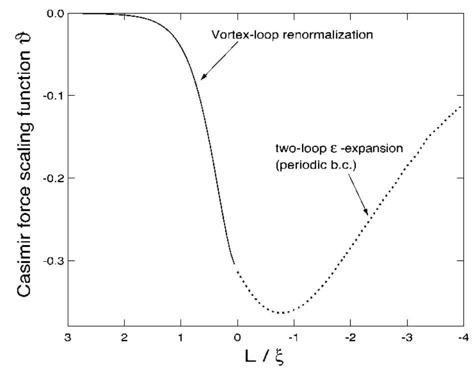

Fig. 3. Casimir force scaling function ϑ versus L/ξ.

perfluid density. Figure 1 shows an evaluation of Eq. 1 using the recursion relations for K_r in Ref. [4], for several reduced temperatures near T$_c$. Very close to T$_c$ there is a crossover from exponential to algebraic decay in the film thickness L, since at T$_c$ a universality condition is $K_r = D_o\,(a_o/a)$ where $D_o = 0.3875$ is a universal constant [4,5]. Inserting this into Eqs. 1 and 2 gives $\delta F/k_BT = \Delta/L^2$ precisely at T$_c$, where $\Delta = -1/(4\pi^2 D_o\beta^3)$. This is exactly the form predicted by Fisher and DeGennes [7] from scaling arguments, where the universal constant Δ is known as the Casimir amplitude. With $\beta = 0.75$ we find $\Delta =$ -0.155, in very reasonable agreement with the value $\Delta =$ -0.20 found in the ϵ-expansion results for periodic boundary conditions [3]. Our calculation is equivalent to periodic boundary conditions since we assume the superfluid density is a constant across the film, with no variation at the wall or free surface. Figure 2 shows the free energy scaling function $\Theta = L^2\,\delta F\big/k_BT$, plotted as a function of L/ξ where ξ is the bulk correlation length. The solid line is the vortex-loop result from evaluating Eq. 1 and $\xi = a_o/K_r$, and where for plotting purposes we have taken ξ positive for T $<$ T$_c$. The dotted curve is the ϵ-expansion result [3] for periodic boundary conditions, normalized by the ratio of the Casimir amplitudes, 0.155/0.20. It is clear there is very good agreement between the two calculations, with the finite slope at T$_c$ matching well.

The Casimir force K_c leading to the film thinning is the derivative of the free energy difference, and is most conveniently written [3] in terms of a scaling function ϑ, $K_c = -\partial\,\delta F/\partial L = k_BT\vartheta/L^3$. Figure 3 shows the results for the scaling function, which can be extracted from the experimental data as in Ref. [2]. This cannot be directly compared with the experimental results, however, since the superfluid density in a real film falls to zero at the boundaries, and hence Dirichlet boundary conditions rather than periodic should be applied. The depression of the superfluid density at the surfaces also has the effect of shifting the superfluid transition temperature downward from the bulk critical temperature T$_\lambda$ by an amount dependent on L, T$_c =$ T$_c$(L). It should be possible in further work with the vortex-loop theory to account for the depressed superfluid density by calculating the excess loop density near a wall [8].

A further effect which should be considered is the crossover to the Kosterlitz-Thouless transition. In the above calculation the iterations were stopped when the loop size reached the film thickness, but actually at that point the loops intercept the boundaries and turn into vortex pairs at longer length scales [8]. Calculations in progress show that this gives a significant contribution to the Casimir force, with the dip shifted down in temperature from T$_\lambda$, and most of it occurring in the nonsuperfluid region above T$_{KT}$. An important experimental check will be to measure the superfluid density simultaneously with the film thinning and verify that T$_{KT}$ is coincident with the onset of thinning.

Acknowledgements

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