Quantum interference and inelastic scattering in a which–way device

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Abstract

A which–way device is one which is designed to detect which of 2 paths is taken by a quantum particle. One such device is represented by an Aharonov–Bohm ring with a quantum dot on one branch. A charged cantilever or spring is brought close to the dot as a detector of the presence of an electron. In this paper we show that, contrary to popular belief, it is in fact possible to change the state of the oscillator while preserving the quantum interference phenomenon, but that this tells us little about the path traversed by the particle.

Key words: NEMS; mesoscopics; Aharonov–Bohm; which–way device

The concept of a "which–way" device has always played an important role in our understanding of quantum mechanics, representing as it does one of the most difficult concepts in modern physics: Schrödinger's famous cat is neither alive nor dead. Textbooks of quantum mechanics typically discuss electrons going through a double slit and state that any attempt to identify which slit the electron goes through will result in the destruction of the interference pattern associated with the double slit.

Based on a recent experiment[1], Armour and Blencowe[2] recently discussed the device illustrated in figure 1. This is based on an Aharonov–Bohm ring[3,4] but with the addition of a quantum dot in one arm of the ring and a charge attached to a microscopic cantilever close by. In the conventional description of this device the electron travels up the device using either the left or the right side of the ring and there is interference at the top junction depending on the relative lengths of the 2 paths and the magnetic flux through the ring. If there is any interaction between the electron and the cantilever the state of the cantilever will be changed and the Aharonov–Bohm interference lost. This is the conventional view. The possibility of coherent coupling to the cantilever may modify this behaviour however. Nevertheless, the results of Armour and Blencowe[2] are essentially consistent with the conventional expectation.

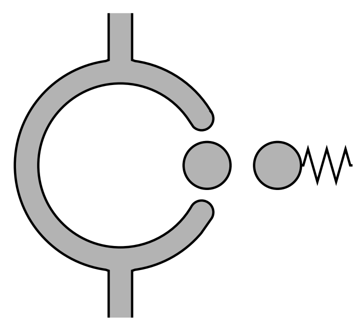

Fig. 1. An Aharonov–Bohm ring containing a quantum dot in close proximity to another charged dot attached to a spring or cantilever.

In this paper we consider other processes on the ring by a numerical simulation. The system may be described by a Hamiltonian of the form

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$$H = -E_\mathrm{e}\left(\frac{\partial}{\partial x_\mathrm{e}} - e\frac{\Phi}{2\pi R}\right)^2 - E_\mathrm{p}\left(\frac{\partial^2}{\partial y_\mathrm{p}^2} - \tfrac{1}{4}y_\mathrm{p}^2\right) + V(x_\mathrm{e}, y_\mathrm{p}) \qquad (1)$$

where x_e and y_p are the electron and phonon coordinates respectively, Φ and R are the magnetic flux and the radius of the ring, and $V(x_\mathrm{e}, y_\mathrm{p})$ is zero except at the quantum dot. It contains a double barrier with a well whose depth depends linearly on y_p to represent the coupling to the cantilever. At the junctions amplitude and current conservation are ensured by appropriate boundary conditions

$$\psi_1 = \psi_2 = \psi_3$$
$$\left.\frac{d\psi}{dx}\right|_1 + \left.\frac{d\psi}{dx}\right|_2 + \left.\frac{d\psi}{dx}\right|_3 = 0$$

where the derivatives are all defined towards the junction.

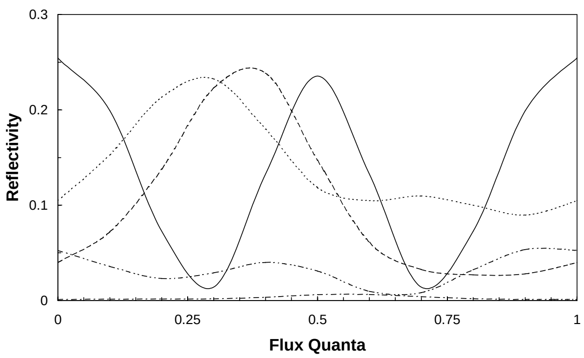

Fig. 2. Reflection coefficients for a single energy as a function of flux. Incident state with 2 phonons, reflected state with 0 (−·), 1 (−−), 2 (—), 3 (· · ·), 4 (− · ·) phonons.

Figure 2 shows the reflectivity from a system with the cantilever initially in the 2 phonon state as the flux is changed by a single flux quantum. The continuous curve, which corresponds to no change in the state of the cantilever, clearly contains components with period of 1 and $\frac{1}{2}$ a flux quantum as expected. The former is the usual Aharonov–Bohm effect[3] while the latter is the correlated back scattering or weak localisation contribution[5] corresponding to interference between paths round the whole ring in clockwise and anticlockwise directions respectively.

The curves corresponding to scattered waves with $n = 3$ and $n = 1$ have comparable amplitudes however. These correspond to processes in which a phonon has been created or annihilated respectively: a change in the state of the cantilever. The simplest process with a period of 1 flux quantum involves interference between 2 waves at the quantum dot. As the coupling to the cantilever depends on the electron density at the dot, this coupling is modulated by the interference between the clockwise and anticlockwise waves. A period $\frac{1}{2}$ results when waves scattered by the dot in opposite directions then interfere at the entrance or exit of the ring.

Why do these processes not destroy the quantum interference? Unlike most inelastic scattering processes considered in a similar context the coupling is to a single vibrational mode rather than to a continuum of such modes. Of course the processes which destroy the conventional Aharonov–Bohm effects will still be operative here and will tend to suppress higher order processes involving multiple circuits of the ring. The similarity of these processes to the simple Aharonov–Bohm effects indicates that they should be observable under similar circumstances.

The device in figure 1 might be improved by combining the cantilever and the dot into one, forming a quantum shuttle[6].

We have shown that the device discussed here, which is a simple modification of an Aharonov–Bohm ring designed as a which–way device, has unexpected properties. It is possible to change the state of the detector, the cantilever, while retaining the quantum interference. This appears to contradict the statement contained in many textbooks that any attempt to determine which path the electron has taken will result in the destruction of the interference. However, the nature of the process involved is such that it tells us absolutely nothing about which path the electron has taken. In fact it confirms that the electron has taken both paths. Hence the conventional view is not wrong; it requires a more subtle interpretation.

Acknowledgements

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