Current sensing noise thermometry from 4.2 K to below 1 mK using a DC SQUID preamplifier

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Abstract

We are using a low-T_c DC SQUID to perform current sensing noise thermometry, by measuring the thermal noise currents in a copper resistor. The temperature is obtained from the Nyquist formula. This is a practical thermometer for use from 4.2 K to below 1 mK, with a percentage precision independent of temperature. Using a 0.34 mΩ resistor, the thermometer had an amplifier noise temperature T_N of 8 μK. A precision of 1.5% was obtained in 200 s. The thermometer was in good agreement with the PLTS-2000 ^{3}He melting curve scale down to 4.5 mK. We have cooled the thermometer successfully below 1 mK, achieving a minumum electron temperature of 300 μK.

Key words:
Noise thermometry; millikelvin temperatures; DC SQUID

1. Introduction

We are developing a thermometer, which is both fast and convenient to use and which provides temperature over a range of more than four orders of magnitude below 4.2 K. A low-T_c DC SQUID is used to measure the thermal noise currents in a copper resistor. The temperature is then obtained from the Nyquist formula. If careful measurements of the value of the resistor and the system gain are made then the thermometer is in principle absolute. In practice, however, it is simpler to use it as a secondary thermometer. In this case only one fixed point at a convenient temperature, for example the boiling point of liquid helium, is necessary in order to calibrate the gain. Alternatively a superconducting fixed point can be incorporated into the thermometer for gain calibration. A detailed report of the work which is summarized here is given in ref [1], which includes a more complete reference to the work of other groups.

This method of noise thermometry was originally introduced by Giffard *et al.* [2,3]. DC SQUIDs currently available are much more sensitive than the RF SQUIDs traditionally used for current sensing noise thermometry. This results in significant improvements in the speed of measurement. In addition the thermometer can be operated to lower temperatures. Thus a good, practical thermometer can be made using commercially available DC SQUIDs.

2. Experimental Details and Potential Performance

A schematic diagram of the experimental set-up is shown in Fig. 1. A resistor R is connected to the input coil of a DC SQUID using a shielded superconducting twisted pair. The DC SQUID is located in the inner vacuum can and held at fixed temperature (in most

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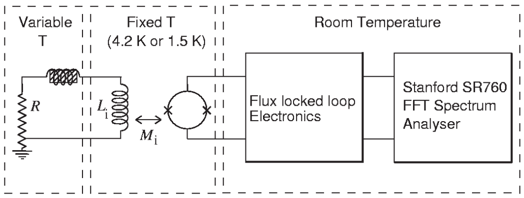

Fig. 1. Schematic diagram of the current sensing noise thermometer. One end of the noise resistor is electrically grounded ensuring adequate cooling of electrons. A fixed point device is incorporated into the SQUID input circuit.

cases 4.2 K) and the resistive sensor is connected to the thermal reservoir whose temperature T is to be measured. The mean square noise current flowing in the SQUID input coil per unit bandwidth, arising from thermal noise in the resistor, is given by

$$\langle I_{\mathrm{N}}^2 \rangle = \frac{4k_{\mathrm{B}}T}{R}\left(\frac{1}{1+\omega^2\tau^2}\right), \tag{1}$$

where k_{B} is Boltzmann's constant and $\omega = 2\pi f$. The resistance R can be taken to be independent of frequency f for our geometries and frequencies. The time constant $\tau = L_{\mathrm{T}}/R$, where $L_{\mathrm{T}} = L_{\mathrm{i}} + L_{\mathrm{s}}$. Here L_{i} is the input coil inductance of the DC SQUID and L_{s} is any additional inductance in the input circuit. L_{s} may be stray inductance associated with the superconducting twisted pair connecting the resistor to the SQUID input coil. In some sensors we incorporated a fixed point device into the input circuit as shown in Fig. 1, and described below [4]. In this case L_{s} can be more significant. The SQUID is operated in flux-locked loop mode. The gain is therefore determined solely by the value of the feedback resistor and the mutual inductance between the feedback coil and the SQUID and is consequently very stable. The output of the flux-locked loop electronics is fed to a Stanford SR760 spectrum analyser, controlled by a PC running LabVIEW. The data are then fit to equation (1), after subtracting a background white noise power, in order to extract the temperature. The time constant τ is also obtained from the fit, enabling any significant temperature dependence of R to be detected.

Equation (1) gives the noise power coming from the resistive sensor at temperature T. The SQUID also produces noise, which can be parameterized by the coupled energy sensitivity ε_{c} [5]. This is the energy equivalent of the minimum detectable current in the input coil, which in terms of the flux noise per $\sqrt{\mathrm{Hz}}$ in the SQUID, $\langle \phi_{\mathrm{N}}^2 \rangle^{1/2}$, is given by

$$\varepsilon_{\mathrm{c}} = \frac{1}{2} L_{\mathrm{i}} \langle I_{\mathrm{N}}^2 \rangle_{\mathrm{sq}} = \frac{1}{2} L_{\mathrm{i}} \frac{\langle \phi_{\mathrm{N}}^2 \rangle}{M_{\mathrm{i}}^2}, \tag{2}$$

where M_{i} is the mutual inductance between the input coil and the SQUID and $\langle I_{\mathrm{N}}^2 \rangle_{\mathrm{sq}}^{1/2}$ is the current in the SQUID input coil necessary to produce a flux of $\langle \phi_{\mathrm{N}}^2 \rangle^{1/2}$in the SQUID. We define the amplifier noise temperature T_{N} to be the temperature at which the resistor would have to be cooled in order that its thermal noise current at zero frequency is equal to $\langle I_{\mathrm{N}}^2 \rangle_{\mathrm{sq}}^{1/2}$. We obtain from equations (1) and (2)

$$T_{\mathrm{N}} = \left(\frac{\varepsilon_{\mathrm{c}}}{2k_{\mathrm{B}}}\right)\left(\frac{R}{L_{\mathrm{i}}}\right) = \frac{\varepsilon_{\mathrm{c}}}{2k_{\mathrm{B}}\tau}\kappa, \tag{3}$$

where $\kappa = 1 + (L_{\mathrm{s}}/L_{\mathrm{i}})$. This is clearly determined by ε_{c}, L_{i} and R. The speed of the thermometer is also determined by the time constant τ, and hence by R. An order of magnitude estimate for the precision obtainable in a given measuring time t_{meas} is given by [1]

$$\frac{\Delta T}{T} = \left(\frac{2\tau}{t_{\mathrm{meas}}}\right)^{1/2} = \left(\frac{2L_{\mathrm{T}}}{t_{\mathrm{meas}}R}\right). \tag{4}$$

The percentage precision in a given measuring time is independent of temperature so long as $T \gg T_{\mathrm{N}}$. Commercially available DC SQUIDs have ε_{c} around 500 h, where h is Planck's constant, compared with around $10^5 h$ for the RF SQUIDs used previously for current sensing noise thermometry. The resistance required for a given T_{N} is therefore much larger when using a DC SQUID, so the measuring time necessary for a given precision is much smaller.

Equations (3) and (4) can be combined to give a figure of merit for the thermometer [6], $t_{\mathrm{meas}} T_{\mathrm{N}} \sigma^2$,which relates the measurement time for a given precision and the amplifier noise temperature. We can write

$$t_{\mathrm{meas}} T_{\mathrm{N}} \sigma^2 = \frac{\varepsilon_{\mathrm{c}}}{k_{\mathrm{B}}}\kappa, \tag{5}$$

where $\sigma = (\Delta T/T)$. There is a trade off between speed and minimum operating temperature, the value of R being set by the minimum operating temperature required. As an example we consider a thermometer suitable for use with a dilution refrigerator of base temperature 5 mK. A noise temperature of 50 μK would ensure that the contributions of SQUID flux noise to the total noise power at zero frequency would be 1 % at base temperature and less than 1 % at higher temperatures. Using equation (5) and putting $\varepsilon_{\mathrm{c}} = 500$ h and $\kappa = 1$, we obtain $t_{\mathrm{meas}} = 4.8$ s for 1 % precision ($\sigma = 0.01$). This would require an 8 mΩ resistance for the SQUID we are using, which has a 1.9 μH input coil inductance. 960 s would be required for this measurement using an RF SQUID with $\varepsilon_{\mathrm{c}} = 10^5$ h. Using a DC SQUID preamplifier much lower minimum temperatures are in principle possible than with the RF SQUID. When operating a thermometer down to 100 μK a T_{N} of 1 μK is desirable. In this case 240 s would be required for 1 % precision using the DC SQUID, whereas the RF SQUID would require 13 hours.

3. Cooling and Shielding the Resistive Sensor

A potential obstacle to developing a noise thermometer for ultra-low temperature work is adequate thermalization of the sensor, dominated by the problem of hot electron effects. At low temperatures, in the presence of a small heat leak the electron temperature T_{e} is higher than the phonon temperature T_{p}. The electrons in a metal cool by transferring energy to phonons at a rate given by

$$P = \Sigma V \left(T_{\mathrm{e}}^5 - T_{\mathrm{p}}^5\right), \tag{6}$$

where V is the volume of the metal. The minimum achievable electron temperature in the presence of a small heat leak to the electrons T_{min} is given by setting T_{p} to zero. Using the value of 1.0×10^{-9} W m^{-3} K^{-5}for Σ obtained on bulk copper [7], and a value for V typical of our experiments (10 mm $\times$ 5 mm $\times$ 25 μm) gives a value for T_{min} of 3.8 mK for a 1 pW heat leak. In our sensors we significantly reduce this problem by electrically grounding one end of the resistor to the relatively large volume copper plate, whose temperature is to be measured, as shown schematically in Fig. 1 [4]. This ensures that the electrons in the resistor are cooled by conduction through the electron system to this reservoir.

We have built a number of sensors, using different sensing resistors and geometries, during the course of this work. All resistors have been cut from copper foil 25μm thick and of 99.9% purity. We chose copper since the electrical resistance of the pure metal is practically independent of temperature at low temperatures [3]. Fig. 2 shows a typical sensor design. The noise resistor is housed in its own niobium shield and niobium screw terminals are used to make connection to the SQUID input coil straightforward. The resistor is mounted on a MACOR base and electrically grounded to a copper plate. This design shows how a fixed point device can be incorporated into the input circuit. Full details together with a preliminary evaluation of this technique are given in ref [1]. The device provides an *in situ* check on the gain of the SQUID amplifier system. A small cylindrical tube of a reference metal, whose superconducting transition temperature T_{c} is well defined, is thermally grounded to the copper plate - the connection is not shown in the Fig. 2. A coil of superconducting wire is wound around the metal and is connected in series with the sensor resistor and the input coil of the SQUID. The inductance of the coil is chosen to be of order L_{i} when the metal is in the normal state. An abrupt drop in the total input inductance occurs on cooling through the superconducting transition and a consequent drop in τ is observed. The DC value of the current noise, from which the temperature is derived, is unaffected by the transition.

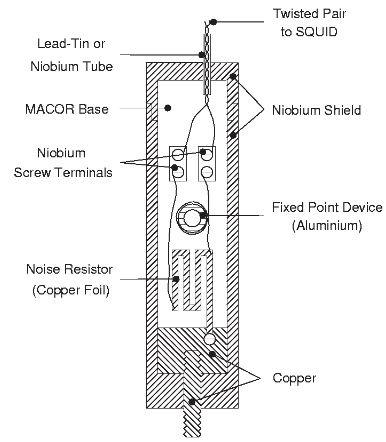

Fig. 2. Diagram of the sensor. See text for details.

In initial tests described in ref [1] we used the noise thermometer as an absolute thermometer at 4.2 K. Here the resistance value and SQUID gain were measured prior to putting together the thermometer. The measured temperature was in agreement with the ITS-90 scale to better than 1 %. We also experimented with the use of a sensor resistor of relatively high resistance (3.99 mΩ), in order to produce a fast thermometer. Here 2 % precision was achieved in 10 s, in approximate agreement with equation (4). In the remainder of this article we describe the performance of a thermometer using a smaller value of sensor resistor, chosen for operation down to below 1 mK.

4. Performance of the current sensing noise thermometer to below 1 mK

We have tested the current sensing noise thermometer on a nuclear demagnetization cryostat. The sensor was similar to that in Fig. 2, however the fixed point device was omitted. The thermometer was used as a secondary thermometer, the gain being calibrated with the fridge stabilized at 100 mK. The temperature at this point was measured with a ^{3}He melting curve thermometer, using the new Provisional Low Temperature Scale, PLTS-2000 [8], to define the temperature. The practical limitations of the present thermometer were investigated down to the lowest temperatures by comparison with a platinum NMR thermometer. A 0.34 mΩ sensor resistance was used, corresponding to T_{N} of 1.9 μK from equation (3), and a measuring time of 124 s for 1% precision from equation (4).

The SQUID (a commercial low-T_{c} DC SQUID from Quantum Design, with ε_{c} = 500 h, L_{i} = 1.9 μH and M_{i} = 10.4 nH) was operated in the vacuum can, mounted and heat sunk to the 4.2 K flange. An in-

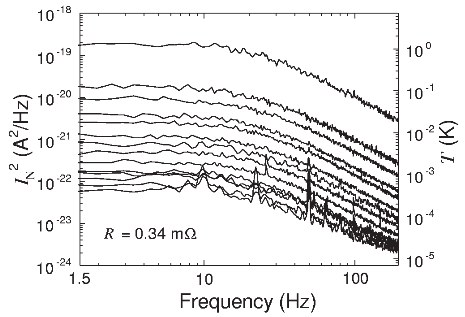

Fig. 3. Noise spectra taken in the region from 1 K to 300 μK.

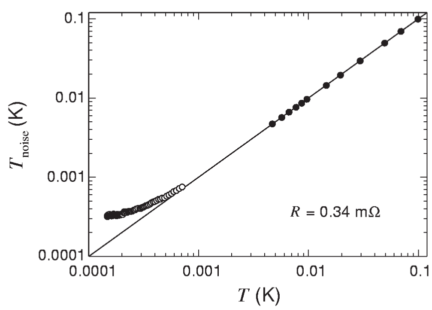

Fig. 4. Temperature obtained from the current sensing noise thermometer versus that obtained from a ^{3}He melting curve thermometer (full circles) and a platinum NMR thermometer (open circles)

termediate niobium junction box containing niobium screw terminals was used, thermally connected to the mixing chamber. Twisted pairs of unclad NbTi wire in 1 mm O.D. niobium tubing were used to connect the junction box to both the SQUID and the sensor.

The thermometer was tested from 4.2 K to below 1 mK. Fig. 3 shows a set of noise spectra taken from ∼ 1 K down to the lowest temperatures. Measurements of noise spectra from 500 Hz to 1 kHz show an additional white noise level, which was found to depend on the final demagnetization field, and which gave a value for T_{N} of 8 μK for the 45.5 mT field in which the spectra in Fig. 3 were taken. This level is a factor of three higher in voltage than that expected from the SQUID alone. This additional noise power was subtracted from all the noise power spectra before fitting to equation (1) to extract the temperature given by the noise thermometer, defined as the Johnson noise temperature T_{noise}.

The Johnson noise temperatures are shown in Fig. 4 at 100 mK and below. The full circles show a comparison between the current sensing noise thermometer and a ^{3}He melting curve thermometer using the PLTS-2000 scale, with the heat switch to the stage closed and the temperature stabilized. Agreement with the PLTS-2000 is within 1% down to 4.7 mK.

The lower temperature data (open circles in Fig. 4) show a comparison with a platinum NMR thermometer. The gain of the pulsed NMR spectrometer used to obtain the platinum NMR temperatures was calibrated against the PLTS-2000 at temperatures above 5 mK. The data were taken whilst slowly warming (over a period of 4.5 days) following a demagnetization to a 45.5 mT. At temperatures above 1 mK the warming rate was too fast to ensure thermal equilibrium. It is clear from the data that at the lowest temperatures the noise thermometer reads hotter than the platinum. The data obey the functional form $T_{\mathrm{noise}}^2 - T^2 = \mathrm{const}$, as expected for a temperature independent heat leak to the sensor, which is being cooled by a pressed metallic contact. The least squares fit to the data is also shown. The intercept corresponds to a minimum T_{noise} of 287 μK for the present thermometer. If this temperature dependence remains valid at higher temperatures then the noise thermometer would read 40 μK above the true temperature at 1 mK and 8 μK hotter at 5 mK. For this thermometer 200 s was required for 1.5 % precision.

Further measurements are required in the range between 1 and 4.5 mK to make preliminary assignments of the ^{3}He fixed points (the 'A' transition and the solid ordering transition). A reduction of the heat leak to the sensor is necessary. Work is also underway on a simpler temperature read-out system.

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