Quasi-Particle density of states of disordered d-wave superconductors

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Abstract

We present a numerical study of the quasi-particle density of states (DoS) of two-dimensional d-wave superconductors in the presence of disorder. We find qualitatively different behavior for smooth and short-ranged disorder. In the former case, we find power law scaling of the DoS with an exponent depending on the strength of the disorder and the superconducting order parameter in quantitative agreement with the theory of Nersesyan *et al.* (Phys. Rev. Lett. **72**, 2628 (1994)). For strong disorder, a qualitative change to an energy independent DoS occurs. In contrast, for short-ranged disorder of sufficient strength, we find localization and derive the dependence of the localization length on the disorder strength from the system size dependence of the micro gap in the DoS near zero energy.

Key words: d-wave superconductor; disorder; localization; density of states

The distinguishing feature of d-wave superconductors as compared to their more conventional s-wave brethren, is the existence of four zero energy 'nodes' on the Fermi surface. The relativistic quasi-particle excitations existing near these nodes determine the low energy transport and thermodynamic properties of the d-wave superconductors. While for a full understanding of these properties self-consistent theories are necessary, we study in the present contribution the most simple, non-selfconsistent system. A further motivation for doing so, is that this system realizes two of the new universality classes [1]. In particular, we are interested in the influence of different kinds of disorder. In contrast to many other disordered systems, the details of disorder are not irrelevant for global observables in the d-wave superconductors. In particular, it matters whether or not the disorder effectively couples the four different quasi-particle sectors. This coupling in turn depends on the range of the disorder potential.

We consider the lattice quasi-particle Hamiltonian

$$H = \sum_{ij;\sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_{ij} \Delta_{ij} c^\dagger_{i\uparrow} c^\dagger_{j\downarrow} + \text{h.c.}, \qquad (1)$$

with the hopping matrix elements t_{ij}, and order parameter Δ_{ij}. The sums run over points of a two-dimensional square lattice with spacing $a = 1$ and the operators $c^\dagger_{i\sigma}$ create a spin-1/2 particle of spin σ at site i. In the following, we take only into account on-site potentials and nearest-neighbor hopping of strength $t = 1$. The order parameter $\Delta_{ij} = \Delta(\delta_{i,j\pm e_x} - \delta_{i,j\pm e_y})$ has $d_{x^2-y^2}$-symmetry. We study correlated disorder of strength W with a correlation length ξ of 0.1 and 2, respectively. For $\xi = 0.1$ the four Dirac 'nodes' are strongly coupled, while for $\xi = 2$ they are effectively decoupled. We will show that these two limits exhibit qualitatively different behavior.

We first analyze long-ranged disorder ($\xi = 2$). Fig. 1 shows the quasi-particle density of states (DoS) near zero energy. The inset zooms in on the 'micro gap' region with a linear DoS [2]. Since the four nodes are effectively decoupled, the physics of the problem is that of a single disordered Dirac node. Nersesyan *et al.* [3] predict for the DoS $\rho(E)$ of this system,

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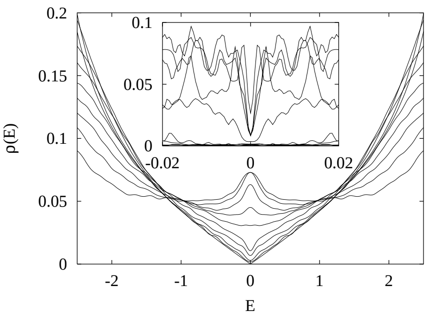

Fig. 1. Density of states for $\Delta = 1$, correlation length $\xi = 2$, and disorder of strength $W = 0, 1, \ldots, 8, 10$ (bottom to top at $E = 0$). The system size is $L = 33$ and the broadening (introduced to as to suppress oscillations on the scale of the mean level spacing) $\Gamma = 0.05$. The inset shows the same data on a smaller scale with $\Gamma = 0.0005$. The finite DoS at $E = 0$ is due to the finite broadening Γ.

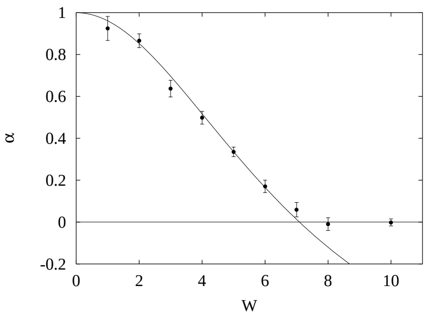

Fig. 2. Exponents α extracted from the fitted curves in Fig. 1 as a function of disorder W for $\Delta = 1$. The solid curve is the result of NTW, eq. (2).

$$\rho(E) \sim |E|^{\alpha}, \qquad \alpha = \frac{1-g}{1+g}, \qquad g = \frac{W^2}{16\pi\Delta t}. \qquad (2)$$

We have fitted power laws to the DoS and show in Fig. 2 a comparison of the numerically obtained exponents α with the prediction of eq. (2) [4]. The agreement is good up to $g = 1$, the limit of applicability of the scaling law. For larger disorder the DoS becomes energy-independent.

Turning now to the case of short-ranged disorder ($\xi = 0.1$), we find a more or less constant DoS with a linear suppression in the micro gap region close to zero energy. It is this micro gap that we concentrate on. The width of the micro gap is given by the mean level spacing of one localization volume or, if the localization length exceeds the system size, by the mean level spacing of the system [5]. As can be seen in Fig. 3, for moderately strong disorder the width of the micro gap first shrinks with increasing system size and eventually saturates when the system size exceeds the localization length. By studying the system size dependence of the micro gap for various disorder strength we can thus extract the dependence of the crossover length scale (localization length) on the disorder strength (Fig. 4).

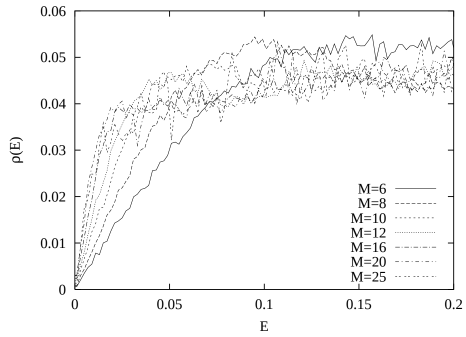

Fig. 3. Density of states for correlation length $\xi = 0.1$, disorder strength $W = 9$ and system sizes $M = 6, \ldots, 25$.

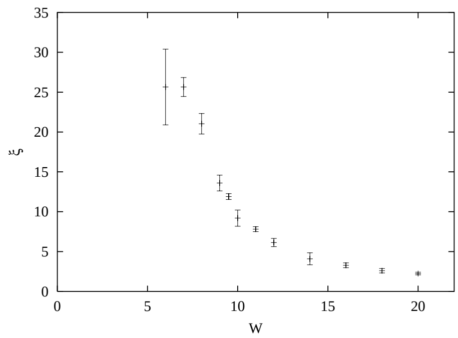

Fig. 4. The crossover length scale (localization length) for short-ranged scatterers ($\xi = 0.1$) as a function of disorder strength W.

To summarize, we have shown that short-ranged and long-ranged disorder leads to qualitatively different behavior in d-wave superconductors. While for long-ranged disorder the quasi-particle density of states shows critical behavior as predicted by Nersesyan *et al.* [3], localization is observed for short-ranged disorder. From the system size dependence of the micro gap we obtained the disorder dependence of the localization length.

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