

# Dynamic Jahn-Teller Effect and Superconductivity in MgB<sub>2</sub>

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## Abstract

We propose a novel mechanism of superconductivity in MgB<sub>2</sub> based on the dynamic Jahn-Teller interactions between the doubly degenerate  $p\sigma$  electronic states and the  $E_{2g}$  phonon modes. The hopping motion of holes in the  $p\sigma$  states of the 2D B layers is constrained by accompanying phonons, and thereby a non-trivial superconducting state with multiple order parameters is found to arise from conventional electron-phonon interactions due to the presence of additional pairing channels.

*Key words:* superconductivity; MgB<sub>2</sub>; multiple order-parameters; Jahn-Teller effect

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Since the discovery of high  $T_c$  superconductivity near 40 K in MgB<sub>2</sub>,[1] experiments and theories have flourished and exhausted various possible issues on the physical properties of the brand-new MgB<sub>2</sub> superconductor. One of the key issues in MgB<sub>2</sub> is about the origin of high  $T_c$  superconductivity, which seems to share little with the previously known Cu-oxide high  $T_c$  superconductors. It naturally led to a revival of interest in the highest  $T_c$  attainable by the conventional electron-phonon mechanism of superconductivity.

Despite such strong evidences in support of electron-phonon mechanism of superconductivity in MgB<sub>2</sub>,[2] however, recent experiments on thermal and spectroscopic properties[3,4] suggested anomalous two-gap superconductivity, which is attributed to the electronic structure of the 2D  $\sigma$ - and 3D  $\pi$ -bands with highly anisotropic electron-phonon couplings[5]. Although the observed superconducting transition temperature of  $\sim$ 40 K by the large electron-phonon coupling is plausible according to several theoretical calculations,[5] however, there still remains some issues on the predicted values of electron-phonon coupling constant  $\lambda$  in comparison with experiments. Further, an important aspect of the electronic structure of MgB<sub>2</sub>, that is, the presence and role of the doubly degenerate

$p\sigma$  electronic states coupled to the doubly degenerate  $E_{2g}$  phonon modes near  $E_F$ , has not been addressed properly. In fact, various physical properties of this coupled degenerate states have been known as the  $E \otimes e$  Jahn-Teller or vibronic system,[6] but its implications on MgB<sub>2</sub> have not been considered yet.

The MgB<sub>2</sub> system consists of B layers with graphite structure and the intercalated Mg ions between the layers[7]. Due to the hybridization of Mg  $s$  and B  $p\pi$  states,[5] the electronic structure of MgB<sub>2</sub> exhibits a unique feature of  $p\sigma$  states crossing the Fermi level  $E_F$ , strongly coupled to the high frequency phonons with  $\omega_{E_{2g}} \approx 70$  meV with large anharmonicity. The doubly degenerate  $p\sigma$  states belong to the  $E_g$  symmetry representation at the  $\Gamma$ -point, where small hole pockets of  $p\sigma$ -bands form almost two-dimensional Fermi surfaces.

The  $E_g$  electron doublet and  $E_{2g}$  phonon doublet states for the single hexagonal B unit cell can be represented by the so-called  $E \otimes e$  Jahn-Teller model[6]. A typical form of the  $E \otimes e$  Hamiltonian can be written by

$$h_0 = \sum_{\alpha=\pm} \omega_0 (a_{\alpha}^{\dagger} a_{\alpha} + \frac{1}{2}) + g\omega_0 \sum_{\alpha=\pm} (a_{-\alpha} + a_{\alpha}^{\dagger}) c_{-\alpha}^{\dagger} c_{\alpha} \quad (1)$$

where  $a_{\alpha}$  and  $c_{\alpha}$  ( $\alpha = \pm$ ) correspond to the phonon doublet and electron doublet states respectively. It is

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well known that one must take account of the dynamical phase of phonons, i.e., non-adiabaticity, in order to get a proper description of this model. In addition, defining a *pseudo-spin* operator by  $J_z = L_z + \frac{1}{2}\tau_z = (n_{a+} - n_{a-}) + \frac{1}{2}(n_{c+} - n_{c-})$  with  $n_{a\alpha} = a_{\alpha}^{\dagger}a_{\alpha}$  and  $n_{c\alpha} = c_{\alpha}^{\dagger}c_{\alpha}$ , one can show that  $[h_0, J_z] = 0$ , that is, the conservation of pseudo-spin. As results, the ground state with a single electron occupancy retains a two-fold degeneracy and acquires a pseudo-spin quantum number  $\tau = \alpha/2 = \pm 1/2$ , implying the coupled motion of electron and phonon with right- or left-handed helicity in the phase space. More detailed characteristics of the ground and excited states of this model have been extensively discussed in the literature[6].

When an electron or hole moves from one site to another, the hopping process is affected by the pseudo-spin configuration at each lattice site. For the electron hopping term

$$H_1 = - \sum_{\alpha \langle ij \rangle \sigma} t_{0i} c_{\alpha i \sigma}^{\dagger} c_{\alpha j \sigma} \quad (2)$$

where  $\sigma$  represents the *real* spin of electrons, one can perform a unitary transformation projecting the ground state on the pseudo-spin configurations of the Jahn-Teller sites. It is noted that  $H_1$  preserves the pseudo-spin symmetry, i.e.,  $[H, J_z] = 0$ . In the weak coupling limit of Eq. (1) for small  $g$ , one can show that the effective pairing-Hamiltonian  $H_{\text{eff}}$  becomes[8]

$$\begin{aligned} H_{\text{eff}} = & \sum_{\alpha \mathbf{k} \sigma} \varepsilon_{\mathbf{k}} c_{\alpha \mathbf{k} \sigma}^{\dagger} c_{\alpha \mathbf{k} \sigma} \\ & - \sum_{\alpha \mathbf{k} \mathbf{k}'} V_{\mathbf{k} \mathbf{k}'}^0 c_{-\alpha \mathbf{k}' \uparrow}^{\dagger} c_{\alpha - \mathbf{k}' \downarrow}^{\dagger} c_{-\alpha - \mathbf{k} \downarrow} c_{\alpha \mathbf{k} \uparrow} \\ & - \sum_{\alpha \mathbf{k} \mathbf{k}'} V_{\mathbf{k} \mathbf{k}'}^1 c_{\alpha \mathbf{k}' \uparrow}^{\dagger} c_{-\alpha - \mathbf{k}' \downarrow}^{\dagger} c_{-\alpha - \mathbf{k} \downarrow} c_{\alpha \mathbf{k} \uparrow} \\ & - \sum_{\alpha \mathbf{k} \mathbf{k}'} V_{\mathbf{k} \mathbf{k}'}^2 c_{\alpha \mathbf{k}' \uparrow}^{\dagger} c_{\alpha - \mathbf{k}' \downarrow}^{\dagger} c_{\alpha - \mathbf{k} \downarrow} c_{\alpha \mathbf{k} \uparrow} \end{aligned} \quad (3)$$

Due to the peculiar nature of the residual electron-phonon interactions with the pseudo-spin symmetry, the hopping process generates additional pairing channels, i.e., three distinct channels contributing to the attractive interactions between electrons due to the exchange of phonons. An analogous residual electron-phonon coupling has been suggested for the case of doped  $C_{60}$  superconductors in terms of the kinematic constraint imposed by the Berry's phase due to the Jahn-Teller coupling at each site[9].

Since the effective pairing Hamiltonian has an extra pairing symmetry apart from the standard BCS model, one may expect a non-trivial gap structure with multiple order parameters. By solving the gap equations including the symmetry breaking hopping terms,[8] we obtained two-gap solutions, which are

responsible for the anomalous behaviors observed in specific heat data[3]. Unlike other theories,[5] both of the gaps are basically isotropic (except for a small anisotropic influence from the  $p\pi$  bands located at a different region of the  $\mathbf{k}$  space). Two s-wave gaps are allowed here owing to the presence of additional quantum numbers for the pseudo-spin. Without pseudo-spins, there would be only one s-wave gap solution giving the lowest free energy. Our model for the dynamic Jahn-Teller mechanism is distinguished from other exotic cases with multiple order parameters or superconducting gaps, which were studied in relation to the heavy fermion systems.[10] It is emphasized that the dynamic Jahn-Teller state is qualitatively different from the usual electronic states with two-fold degeneracy. The pseudo-spin  $J_z$  conservation derived from the Jahn-Teller state acts as a strong constraint in the electron-phonon interactions and pairing interactions as well.

In summary we proposed a mechanism of superconductivity in  $MgB_2$  based on a dynamic Jahn-Teller effect arising from the interplay between the doubly degenerate  $p\sigma$  electronic states and the  $E_{2g}$  phonon modes. The pseudo-spin constraint imposed on the motion of holes opens up additional pairing channels, which contribute to the non-trivial superconducting properties in  $MgB_2$ .

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