A-B Transition of Superfluid ^{3}He in Aerogel Under Magnetic Field

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Abstract

A-B phase transition of superfluid ^{3}He in aerogel under magnetic field is discussed using the homogeneous impurity model. The second order A-B transition temperature $T_{\rm AB}$ is calculated in the whole temperature range as a function of the magnetic field. It is shown that the GL result is correct only in the vicinity of the transition temperature $T_{\rm ca}$ of the liquid ^{3}He in aerogel.

Key words: ^{3}He, Aerogel, Impurity, A-B transition, Magnetic field

1. Introduction

Since the discovery of the superfluidity of liquid ^{3}He in aerogel, there has been considerable interest in impurity scattering effects on p-wave superfluid. Clear evidence of the impurity scattering effect has been observed as the reduction of the superfluid transition temperature and the superfluid density. However, the identification of the order parameter structure and the modification of the phase diagram have not yet been established. In the p-wave spin-triplet pairing systems under magnetic field, A-phase with equal-spin pairs is more favorable than the B-phase that has $\uparrow\downarrow + \downarrow\uparrow$ pairs as well. Such an A-B transition under magnetic field has been observed in the superfluid phase of ^{3}He in aerogel. [1,2] Brussaard *et al.* measured the superfluid density ρ_s in aerogel under magnetic fields using a vibrating wire technique and found a clear signature of A-B transition as an abrupt change in ρ_s. Gervais *et al.* succeeded in detecting multiple phase transitions of the ^{3}He-aerogel system in magnetic fields using high frequency sound.

In this paper we present a theoretical study of impurity scattering effect on the phase diagram under magnetic field. We calculate within the weak coupling theory the A-B transition line over the whole temperature range using the self-consistent order parameter.

2. Formulation

Here, we consider the second order transition from the BW state with d-vector $d_{x,y} = \Delta_\perp \hat{k}_{x,y}$ and $d_z = \Delta_\parallel \hat{k}_z$ to the planer state with $d_{x,y} = \Delta_\perp \hat{k}_{x,y}$ and $d_z = 0$. As long as the energetics of the A-B transition is concerned, it suffices to consider the phase transition between the BW state and the planar state, because the ABM state and the planar state have the same free energy within the weak coupling theory even in the presence of impurities.

We start with the Gor'kov equation including the Zeeman energy and an impurity scattering self-energy determined within the self-consistent t-matrix approximation. To render calculations tractable, we adopt the s-wave scattering approximation. [3] Within this approximation, the Gor'kov equation can be written as

$$\left[i\tilde{\omega}_n - \begin{pmatrix} \xi + \frac{\omega_L}{2}\sigma_3 & \Delta \\ \Delta^\dagger & -\xi - \frac{\omega_L}{2}\sigma_3 \end{pmatrix} \right] \hat{G} = 1, \qquad (1)$$

where $\omega_L = \frac{\gamma B}{1+F_0^a}$ is the Larmor frequency including the Fermi liquid effect, σ_3 is a Pauli matrix in spin space and $\Delta = i(\mathbf{d} \cdot \sigma)\sigma_2$. In the s-wave scattering approx-

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imation, the impurity effects occur only in the renormalization of the Matsubara frequency determined by

$$\tilde{\omega}_n = \omega_n + \frac{1}{2\tau}\frac{\tilde{\omega}_n \langle 1/E_n \rangle}{1+\sigma(\tilde{\omega}_n^2 \langle 1/E_n \rangle^2 - 1)}, \quad (2)$$

where τ is the relaxation time, σ is a normalized scattering cross section ($\sigma \to 0$ corresponds to the Born limit and $\sigma \to 1$ to the unitarity limit) and $E_n = \sqrt{\tilde{\omega}_n^2 + \Delta_\perp^2(\hat{k}_x^2 + \hat{k}_y^2)}$. The bracket $< \cdots >$ means the angle average over the Fermi surface.

Since we are interested in the second order A-B transition, we may use the gap equation linear in $\Delta_\parallel$. After straightforward calculation, we obtain the linearized gap equation in the form

$$\log \frac{T_{ABa}}{T_{ca}} + S_1 = 3\pi T_{ABa} \sum_n \left\langle \frac{\hat{k}_z^2 E_n}{E_n^2 + \frac{\omega_L^2}{4}} \right\rangle . \quad (3)$$

The equal-spin component of the order parameter $\Delta_\perp$ is determined by the gap equation

$$\log \frac{T}{T_{\rm ca}} + S_1 = 3\pi T \sum_n \left\langle \frac{\hat{k}_x^2}{E_n} \right\rangle , \quad (4)$$

where $S_m = \sum_{n=0} 1/(n + \frac{1}{2} + x)^m$ and $x = 1/4\pi T_{\rm ca}\tau$ is the pair-breaking parameter.

3. Numerical results

We calculate the phase diagram using the parameters for 12.8 bar liquid ^{3}He in 98 % aerogel with $T_{\rm ca}$=1.3 mK[4]. In Fig. 1, we show the second order A-B transition line in the Born and unitarity limits and compare the result with that of the pure liquid ^{3}He at the same pressure. The transition temperature is substantially suppressed by the impurity scattering. In the unitarity limit, we find no hump behavior at intermediate temperatures. It may happen that in the unitarity limit there is no first order phase transition between the BW state and the planar state.

Now we examine the behavior of the transition line near the superfluid transition temperature $T_{\rm ca}$. Using the GL expansion, we obtain

$$1 - \frac{T_{\rm ABa}}{T_{\rm ca}} = \frac{S_3 + \frac{5}{3}(\sigma - \frac{1}{2})xS_4}{8(1 - xS_2)} \left[\frac{\gamma B}{\pi T_{\rm ca}(1 + F_0^a)} \right]^2 . \quad (5)$$

In Fig. 2, we plot $T_{\rm ABa}/T_{\rm ca}$ as a function of the square of external magnetic field. It is found that the GL theory is only valid in the temperature range $1 - T_{\rm ABa}/T_{\rm ca} < 0.05$ in this 12.8 bar case. Deviation from the GL theory occurs earlier when the transition temperature $T_{\rm ca}$ is smaller. This is because the contribution from the fourth order term $(\omega_L/T_{\rm ca})^4$ becomes important.

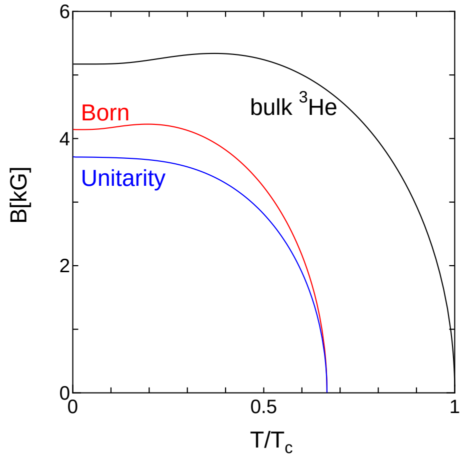

Fig. 1. B-T phase diagram of the superfluid ^{3}He in aerogel at a pressure of 12.8 bar. Parameters are chosen as $T_{\rm ca} = 1.3$mK and $F_0^a = -0.75$ in accordance with Ref. [4].

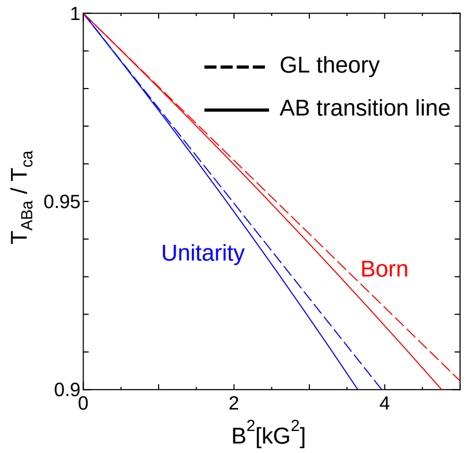

Fig. 2. Comparison with GL theory. $T_{\rm ABa}/T_{\rm ca}$ is plotted as a function of B^2 near the superfluid transition temperature. Solid lines show our numerical results. Dashed lines are the prediction of the GL theory.

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