A decoupling-free solid-state NMR quantum computer

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Abstract

We present a decoupling-free solid-state NMR quantum computer with a switchable controlled-NOT gate installed in a one-dimensional antiferromagnet with a spin gap. The gate switching is realized by a local singlet-triplet excitation across the spin gap, which induces a Suhl-Nakamura-type internuclear coupling between control and target qubits. Naive calculation for the coupling strength with some realistic parameters indicates that the coupling can be strong enough to serve as a controlled-NOT gate.

Key words: quantum computer; NMR; one-dimensional antiferromagnet

A nuclear spin system provides great promise for a quantum computer (QC), because it enables good isolation of qubits from the environment and excellent control of qubits by the well-established technique of nuclear magnetic resonance (NMR). The NMR-QCs proposed so far can be categorized into two types, i.e., solution[1,2] and solid-state[3,4]. The former is advantageous in terms of decoherence time because quick motions of molecules in solution provide the decoupling effect between qubits, while disadvantageous in terms of scalability because the number of qubits (N) is limited to that in the available nuclei in a molecule. Although N in solution QCs has reached seven qubits to date, one cannot expect it to exceed 100, where the quantum computer is believed to surpass its classical analogue.

A solid-state QC is advantageous in this respect because N is not limited there, but a decoherence issue becomes more serious because of the complicated internuclear couplings in solids. Although these internuclear couplings are available for the controlled-NOT (c-NOT) gate inevitable for a QC, one needs to decouple qubits whenever the c-NOT gate is unnecessary, which is formidable for the complicated couplings in solids. Here, we propose a decoupling-free solid-state NMR-QC with a switchable interqubit coupling[5,6]. The switching is realized by a Suhl-Nakamura (SN) interaction mediated by magnons in a one-dimensional antiferromagnet with a spin gap. A naive numerical calculation for the coupling strength is presented for some realistic parameters.

A schematic illustration of the gate switching is shown in Fig. 1. A QC is installed in an antiferromagnetic electron spin chain, which is placed in a magnetic field gradient and at low T. The spin chain is selected to have a singlet ground state ($\mid ss_z\rangle = \mid 00\rangle$) with a finite gap to the lowest triplet branch of the magnon modes because of some quantum effect. Examples include spin ladder, Haldane and spin-Peierls systems. Suppose that nuclear spins ($I = 1/2$) serving as qubits can be placed periodically, e.g., every $10a$ (a: lattice spacing), and that the rest of the sites are occupied by $I = 0$ nuclei. The field gradient enables access to each individual qubit by tuning the NMR frequency.

At temperatures well beneath the spin gap energy (E_g), all the internuclear couplings are absent. Then, a microwave tuned to E_g is irradiated to the QC, which creates a packet of triplet magnons between the con-

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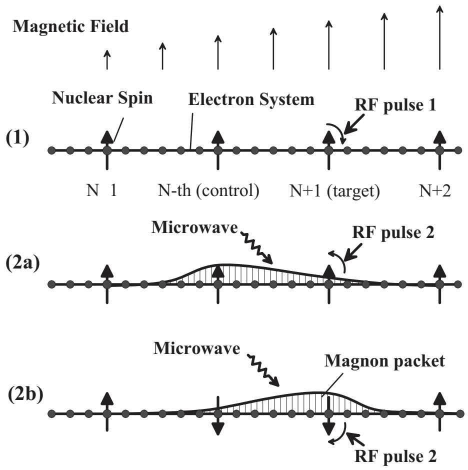

Fig. 1. Schematic illustration of the c-NOT-gate switching. $I = 0$ and 1/2 (= qubits) nuclei are shown by circles and arrows, respectively. (1) All the inter-nuclear couplings are switched off in the absence of magnons. (2) A magnon packet (hatched part) is excited between the control (I_N) and target (I_{N+1}) qubits. The magnon packet couples the two qubits.

trol and target qubits where the c-NOT gate is to be performed. The packet switches on the Suhl-Nakamura type internuclear coupling between the qubits. Note that E_p at each part of the QC is uniquely given in the magnetic field gradient.

The SN interaction between the qubits is given by $H_{\rm SN} = W_{ij} I_i^z I_j^z$ with [7],

$$W_{ij} = \left(\frac{\gamma_n A}{N}\right)^2 \sum_{k \neq k'} \frac{n_k - n_{k'}}{\epsilon_{k'} - \epsilon_k} \cos\{(k - k')r_{ij}\}. \qquad (1)$$

Here, n_k and ϵ_k are, respectively, the population and the energy of the magnon with the wave number k, and r_{ij} is the distance between the two qubits of interest. A and N are the hyperfine coupling constant and the number of sites (including the $I = 0$ sites) inside the packet, respectively.

Suppose the magnon dispersion of the spin ladder, as an example, in the form [8], $\epsilon(k_n) = C + J(j_1 - j_1^3/4)\cos(k_n) + ...$, where $k_n = n\pi/N$, $j_1 = J_1/J$ with J_1 and J being the intra- and inter-chain exchange interactions, and C is the part independent of k_n. Since $n(k) = 0$ for $k \neq 0$, W_{ij} can be calculated as,

$$W_{ij} = \frac{2\gamma_n^2 A^2 \{n(0)/N\}}{J(j_1 - \frac{1}{4}j_1^3)} \cdot \frac{1}{N} \sum_{n=1}^{N} \frac{\cos(k_n r_{ij})}{\cos(k_n) - 1}$$
$$\equiv X \cdot F(r_{ij}/a, N). \qquad (2)$$

where $X \equiv 2\gamma_n^2 A^2\{n(0)/N\}/J(j_1 - j_1^3/4)$. Assuming A = 100 kOe/μ_B, $\gamma_n/(2\pi) = 4.3$ MHz/kOe (in the case of ^1H), $J = 50$ K, $j_2 = 0.2$ and $n(0)/N = 0.01$, one obtains $X = 17$ kHz. On the other hand, the range function $F(r_{ij}/a, N)$ in Eq. (2) can be calculated as

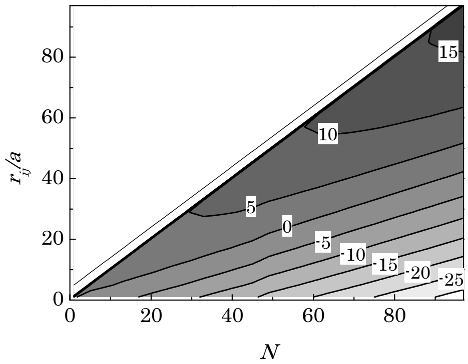

Fig. 2. Contour plot of the range function $F(r_{ij}/a, N)$ in eq. (2), where r_{ij} is the distance between control and target qubits, and N is the number of nuclei inside the triplet packet.

shown in Fig. 2, which is in the range between -30 and 20. In comparison with the nuclear dipole couplings in solids ($\sim$ 10 kHz) [4,9] and the J-couplings in solution NMR-QCs ($10^1 \sim 10^3$ Hz) [1,2], one finds that the SN coupling can be strong enough to serve as a quantum gate.

In conclusion, we presented a decoupling-free quantum computer with a magnon-mediated quantum gate installed in a 1-D antiferromagnet. A simple order estimation indicates that the coupling strength can be strong enough to serve as a controlled-NOT gate.

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