Cooper-pair-box qubits in a quantum electrodynamic cavity

J. Q. You [a,1], Franco Nori [a,b]

[a] *Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi 351-0198, Japan*

[b] *Center for Theoretical Physics, Physics Department, Center for the Study of Complex Systems, University of Michigan, Ann Arbor, MI 48109-1120, USA* [2]

Abstract

We study the quantum dynamics of Cooper-pair-box qubits in a quantum electrodynamic cavity. We show the existence of Rabi oscillations for both single- and multi-photon processes and propose a quantum computing scheme.

Key words: superconducting Josephson charge qubits; quantum information processing

1. Introduction

Because of possible scalability to large-scale networks using modern microfabrication techniques, considerable efforts have recently been devoted to solid-state qubits. Proposed solid-state architectures include those using electron spins in quantum dots [1,2] and Josephson-junction devices [3,4]. Here we show that the coupled system of a Cooper-pair box with a superconducting loop and the microwave mode in a quantum electrodynamic cavity undergoes Rabi oscillations and propose a new quantum computing scheme in which any two qubits (not necessarily neighbors) can be coupled through photons in the cavity. In sharp contrast to the usual Jaynes-Cummings model, multi-photon processes are shown in the Josephson charge qubit. As revealed by the very recent experiment on Rabi oscillations in a Cooper-pair box [5], these multi-photon processes may be important in the charge-qubit system. The dynamics of a Josephson charge qubit coupled to a quantum resonator was studied in [6], but the model involves: (a) only one charge qubit, (b) only the Rabi oscillation with a single excitation quantum of the resonator, and (c) no quantum computing scheme.

2. Model Hamiltonian

The Josephson charge qubit is achieved in a Cooper-pair box [7]. We study the Cooper-pair box with a SQUID loop pierced by a flux Φ_X (see the schematic illustration in [3]). In this structure, the superconducting island with excess charge $Q = 2ne$ is coupled to a segment of a superconducting ring via two Josephson junctions (each with capacitance C_J and Josephson coupling energy E_J^0). Also, a voltage V_X is coupled to the superconducting island through a gate capacitor C. In the spin-$\frac{1}{2}$ representation with charge states $|\uparrow\rangle = |n\rangle$ and $|\downarrow\rangle = |n+1\rangle$, the Hamiltonian of the system can be reduced to $H = \varepsilon(V_X)\sigma_z - \frac{1}{2}E_J(\Phi_X)\sigma_x$ in the charging regime [3], where $\varepsilon(V_X) = 2E_c[CV_X/e - (2n+1)]$, and $E_J(\Phi_X) = 2E_J^0\cos(\pi\Phi_X/\Phi_0)$, with $E_c = e^2/2(C + 2C_J)$, and $\Phi_0 = h/2e$. This single-qubit Hamiltonian has two eigenvalues $E_\pm = \pm\frac{1}{2}E$, with $E = [4\varepsilon^2(V_X) + E_J^2(\Phi_X)]^{1/2}$, and eigenstates $|e\rangle = \cos\xi|\uparrow\rangle - \sin\xi|\downarrow\rangle$, and $|g\rangle = \sin\xi|\uparrow\rangle + \cos\xi|\downarrow\rangle$, with $\xi = \frac{1}{2}\tan^{-1}(E_J/2\varepsilon)$. Using these eigenstates as new basis, the Hamiltonian becomes $H = \frac{1}{2}E\rho_z$, where $\rho_z = |e\rangle\langle e| - |g\rangle\langle g|$. Here we employ $\{|e\rangle, |g\rangle\}$ to represent the qubit.

When a nonclassical microwave field is applied, the total flux Φ through the SQUID loop is a quantum variable, $\Phi = \Phi_X + \Phi_f$, where Φ_f is the microwave-field-induced flux through the SQUID loop. Here we assume that a single-qubit structure is embedded in a

[1] Corresponding author. E-mail: jqyou@postman.riken.go.jp

[2] Permanent address.

QED microwave cavity with only a single photon mode λ. The vector potential of the nonclassical microwave field can be written as $\mathbf{A}(\mathbf{r}) = \mathbf{u}_\lambda(\mathbf{r})a + \mathbf{u}_\lambda^*(\mathbf{r})a^\dagger$, where $a^\dagger(a)$ is the creation (annihilation) operator of the cavity mode. Thus, the flux Φ_f is given by $\Phi_f = \Phi_\lambda a + \Phi_\lambda^* a^\dagger = |\Phi_\lambda|(e^{-i\theta}a + e^{i\theta}a^\dagger)$, with $\Phi_\lambda = \oint \mathbf{u}_\lambda \cdot d\mathbf{l}$, where the contour integration is over the SQUID loop. We shift the gate voltage V_X (and/or vary Φ_X) to bring the single-qubit system into resonance with k photons: $E \approx k\hbar\omega_\lambda$, $k = 1, 2, 3, \ldots$. Expanding the functions $\cos(\pi\Phi_f/\Phi_0)$ and $\sin(\pi\Phi_f/\Phi_0)$ into series of operators and employing the rotating-wave approximation, we derive the total Hamiltonian of the system (including the microwave field)

$$H = \frac{1}{2}E\rho_z + \hbar\omega_\lambda(a^\dagger a + \frac{1}{2}) + H_{Ik}, \quad (1)$$
$$H_{Ik} = \rho_z f(a^\dagger a) + \left[e^{-ik\theta}|e\rangle\langle g|a^k g^{(k)}(a^\dagger a) + \text{H.c.}\right],$$

where the explicit expressions of $f(a^\dagger a)$ and $g^{(k)}(a^\dagger a)$ are given in [8].

3. Multi-photon Rabi oscillations

The eigenvalues of the total Hamiltonian (1) are $\varepsilon_\pm(l,k) = \hbar\omega_\lambda[l + \frac{1}{2}(k+1)] + \frac{1}{2}[f(l) - f(l+k)] \pm \frac{\hbar}{2}\sqrt{\delta_{l,k}^2 + \Omega_{l,k}^2}$, and the corresponding eigenstates are given by $|+, l\rangle = e^{-ik\theta}\cos\eta|e,l\rangle + \sin\eta|g, l+k\rangle$, and $|-, l\rangle = -\sin\eta|e,l\rangle + e^{ik\theta}\cos\eta|g, l+k\rangle$, where $\Omega_{l,k} = 2g^{(k)}(l+k)[(l+1)(l+2)\cdots(l+k)]^{1/2}/\hbar$ is the Rabi frequency, $\delta_{l,k} = (E/\hbar - k\omega_\lambda) + [f(l) + f(l+k)]/\hbar$, and $\eta = \frac{1}{2}\tan^{-1}(\Omega_{l,k}/\delta_{l,k})$.

When the system is initially at the state $|e,l\rangle$, after a period of time t, the probabilities for the system to be at states $|g, l+k\rangle$ and $|e,l\rangle$ are

$$|\langle g, l+k|\psi(t)\rangle|^2 = \frac{\Omega_{l,k}^2}{\delta_{l,k}^2 + \Omega_{l,k}^2}\sin^2\left(\frac{1}{2}\sqrt{\delta_{l,k}^2 + \Omega_{l,k}^2}\,t\right),$$

and $|\langle e,l|\psi(t)\rangle|^2 = 1 - |\langle g, l+k|\psi(t)\rangle|^2$. This is the Rabi oscillation with k photons involved in the state transition. Very recently, Nakamura *et al.* [5] observed the multi-photon Rabi oscillations in the charge qubit. Different to the case studied here, the microwave field was employed there to drive the gate voltage to oscillate. Here, to implement quantum computing, we consider the Cooper-pair box with a SQUID loop and use the microwave field to change the flux through the loop.

4. Quantum computing

Let us consider more than one single charge qubit in the QED cavity, and the cavity initially prepared at the zero-photon state. A controlled-phase-shift gate can be implemented via the following steps:

(i) For all Josephson charge qubits, let $\Phi_X = \frac{1}{2}\Phi_0$, then $\cos(\pi\Phi_X/\Phi_0) = 0$, which yields $f(a^\dagger a) = 0$. Furthermore, the gate voltage for a control qubit, say A, is adjusted to have the qubit on resonance with the cavity mode ($E = \hbar\omega_\lambda$) for a period of time $t = \pi\hbar/2g^{(1)}(1)$ with $\theta = 0$, while all other qubits are kept off-resonance.

(ii) While all qubits are kept off-resonance with the cavity mode and the flux Φ_X is originally set to $\Phi_X = \frac{1}{2}\Phi_0$ for each qubit, we change Φ_X to zero for only the target qubit, say B, during a period of time $t = \pi\hbar/2|f(1) - f(0)|$.

(iii) Qubit A is again brought into resonance for $t = \pi\hbar/2g^{(1)}(1)$ with $\theta = 0$, as in step (i).

(iv) The unitary operation given in step (ii) is applied successively to the control and target qubits with $t = 3\pi\hbar/4|f(0)|$ and $(2\pi - \beta)\hbar/|f(0)|$, respectively, where $\beta = \pi|f(0)|/2|f(1) - f(0)|$.

A sequence of such conditional two-bit gates in combination of one-bit rotations constitute a complete set of logic gates for quantum computing [9].

Acknowledgements

We acknowledge support from the Frontier Research System at RIKEN; the AFOSR, ARDA; and the US National Science Foundation grant No. EIA-0130383.

References

[1] D. Loss, D.P. DiVincenzo, Phys. Rev. A **57** (1998) 120; X. Hu, S. Das Sarma, Phys. Rev. A **61** (2000) 062301 (2000), and references therein; X. Hu, R. de Sousa, S. Das Sarma, Phys. Rev. Lett. **86** (2001) 918.

[2] A. Imamoḡlu *et al.*, Phys. Rev. Lett. **83** (1999) 4204; M.S. Sherwin, A. Imamoḡlu, T. Montroy, Phys. Rev. A **60** (1999) 3508.

[3] Y. Makhlin, G. Shön, A. Shnirman, Nature **398** (1999) 305.

[4] J.E. Mooij *et al.*, Science **285** (1999) 1036; T.P. Orlando *et al.*, Phys. Rev. B **60** (1999) 15398.

[5] Y. Nakamura, Yu. A. Pashkin, J.S. Tsai, Phys. Rev. Lett. **87** (2001) 246601.

[6] A.D. Armour, M.P. Blencowe, K.C. Aschwab, Phys. Rev. Lett. **88** (2002) 148301; O. Buisson, F.W.J. Hekking, cond-mat/0008275; F. Marquart, C. Bruder, Phys. Rev. B **63** (2001) 054514.

[7] Y. Nakamura, Yu. A. Pashkin, J.S. Tsai, Nature **398** (1999) 786.

[8] J.Q. You, F. Nori, preprint.

[9] S. Lloyd, Phys. Rev. Lett. **75** (1995) 346; D. Deutsch, A. Barenco, A. Ekert, Proc. R. Soc. London, Ser. A **449** (1995) 669.