Breathing mode collective excitation of a Bose-Einstein Condensate in a low-dimensional trap

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Abstract

Using a sum-rule method and a time-dependent variational method, we study the breathing mode collective-excitation frequency of a trapped Bose-Einstein condensate. We show that the result for the three-dimensional trap applies also that to the ideal two- or one-dimensional trap, where the axial or radial degree of freedom is completely neglected. In the case of a realistic two- or one-dimensional trap, we obtain the lowest order correction for the collective-excitation frequency due to the finiteness of the trap frequency in the tightly trapped direction. We also show numerical results for the collective-excitation frequencies using parameters relevant to recent experiments.

Key words: Bose-Einstein condensates, breathing mode collective excitation, low-dimensional traps

Bose-Einstein condensates (BEC) in harmonic traps have been of great interest because of their rich many-body phenomena such as collective excitations [1–9], which are significantly affected by the inter-atom interaction. Recently, a two-dimensional (2D) [10] or a one-dimensional (1D) [10–12] BEC has been successfully fabricated and collective excitations in the low dimensional traps have been interested. In this paper, we study the breathing mode ($m = 0$ mode: m is the magnetic quantum number) collective-excitation frequency of a BEC in a 2D or 1D trap within the mean-field theory for the condensate at zero temperature, by employing two analytical methods: a sum-rule method [1,6,7] and a time-dependent variational method [4].

The sum-rule method for the breathing mode collective excitation in an axially-symmetric trap has been introduced in Ref. [6] and [7]. By using the method, we obtain the exact upper bound for the collective-excitation frequency for arbitrary number of atoms as

$$(\omega^{\text{upper}})^2 = 2 + \frac{\lambda^2}{2\langle U_z\rangle}(3\langle U_z\rangle + \langle T_z\rangle) - 2\Big\{1 - \frac{\lambda^2}{2\langle U_\perp\rangle\langle U_z\rangle} \times \big[\langle T_\perp\rangle(\langle U_\perp\rangle - \langle T_\perp\rangle) + 2\langle U_\perp\rangle(\langle U_z\rangle + \langle T_z\rangle)\big] + \frac{\lambda^4}{16\langle U_z\rangle^2}(3\langle U_z\rangle + \langle T_z\rangle)^2\Big\}^{\frac{1}{2}}. \quad (1)$$

where, $\lambda \equiv \omega_z/\omega_\perp$ ($\omega_\perp \equiv \omega_x \equiv \omega_y \equiv 1$) is the asymmetry parameter of an axially-symmetric harmonic trap, $T_{\perp(z)}$ is the x or y (z) component of the kinetic energy, $U_{\perp(z)}$ is the x or y (z) component of the trap potential energy, and $\langle\cdots\rangle$ denotes the mean value per atom over the condensate wave function. Here, we have used the virial theorem [9] such that $\langle H_{\text{int}}\rangle/2 = \langle U_l\rangle - \langle T_l\rangle$ ($l = \perp$ or z), where H_{int} is the inter-atom interaction.

On the other hand, the time-dependent variational method [4], which is based on a Gaussian variational wave function for the condensate, gives an approximate collective-excitation frequency which is the same as Eq. (1) except that the mean value in Eq. (1) is calculated over the Gaussian variational wave function.

Both results reproduce the exact results:

$$\omega = \omega_\perp\sqrt{2 + 3\lambda^2/2 - \sqrt{9\lambda^4 - 16\lambda^2 + 16}/2} \quad (2)$$

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obtained in the Thomas-Fermi (TF) limit $\langle T_\perp \rangle = \langle T_z \rangle = 0$ for a three-dimensional (3D) trap [1], $\omega = 2$ obtained in a purely 2D trap $\langle T_z \rangle = \langle U_z \rangle = \infty$ (ω does not depend on the number of atoms in the purely 2D trap) [2,3], and $\omega = \sqrt{3}\lambda$ obtained in the TF limit $\langle T_z \rangle = 0$ for a purely 1D trap $\langle T_\perp \rangle = \langle U_\perp \rangle = \infty$ [5,8].

One may be aware that the result in the purely 2D (1D) trap is different from $\omega = \sqrt{10/3}$ ($\omega\sqrt{5/2}\lambda$) [1], which is the 2D (1D) limit $\lambda \to \infty$ (0) of Eq. (2) for the 3D trap. Because of the virial theorem, if we first take the TF limit in a 3D trap, the energy scales in the axial and radial direction are comparable even if we take the 2D or 1D limit ($\lambda \to \infty$ or 0) after the TF limit. In this case, by contrast to the purely 2D or 1D case, the condensate can oscillate not only to the weakly trapped direction but also to the tightly trapped direction and can reduce the interaction energy by the out-of-phase oscillation. As a result, the collective-excitation frequency in the 2D or 1D limit of the TF limit for a 3D trap is slightly smaller the frequency in the purely 2D or 1D trap.

Now, let us consider the case with finite ω_z and with large but finite N (N: number of atoms) in low dimensional traps. In the 2D trap with large N, we assume $\langle T_z \rangle$, $\langle U_z \rangle \gg \langle H_{\rm int} \rangle/2 \approx U_\perp \gg \langle T_\perp \rangle$, which is equivalent to $\lambda \gg \sqrt{Na}\lambda^{\frac{1}{4}} \gg 1/(\sqrt{Na}\lambda^{\frac{1}{4}})$ because $\langle U_z \rangle \propto \lambda$, $\langle H_{\rm int} \rangle \propto \lambda^{\frac{1}{4}}\sqrt{Na}$, and $\langle T_\perp \rangle \propto 1/(\lambda^{\frac{1}{4}}\sqrt{Na})$. Up to the first order of $\langle H_{\rm int} \rangle/\langle U_z \rangle$ or $\langle T_\perp \rangle/\langle U_\perp \rangle$, we obtain $\omega^{\rm upper} \approx 2 - \frac{\langle H_{\rm int} \rangle}{16\langle U_z \rangle} \approx 2 - \frac{1}{12}\left(\frac{8}{\pi}\right)^{\frac{1}{4}} \lambda^{-\frac{3}{4}}\sqrt{Na/d}$ by the sum-rule method, where a is the s-wave scattering length in unit of the harmonic oscillator length $\sqrt{\hbar/M\omega_\perp} \equiv 1$ (M is the atomic mass), and $\omega \approx 2 - \langle H_{\rm int} \rangle_G/(16\langle U_z \rangle_G) \approx 2 - \left(\lambda^{-\frac{3}{4}}/4(8\pi)^{\frac{1}{4}}\right)\sqrt{Na}$ by the time-dependent variational method.

In the 1D case, we assume $\langle T_\perp \rangle$, $\langle U_\perp \rangle \gg \langle H_{\rm int} \rangle/2 \approx U_z \gg \langle T_z \rangle$, which is equivalent to $1 \gg \lambda Na \gg \lambda^2/(Na)$ because $\langle U_\perp \rangle \propto 1$, $\langle H_{\rm int} \rangle \propto (\lambda Na)^{\frac{2}{3}}$, and $\langle T_z \rangle \propto [\lambda^2/(Na)]^{\frac{2}{3}}$. Up to the first order of $\langle H_{\rm int} \rangle/\langle U_\perp \rangle$ or $\langle T_z \rangle/\langle U_z \rangle$, we obtain $\omega^{\rm upper} \approx \sqrt{3}\lambda\left[1 + \frac{5}{36}\left(\frac{\lambda}{9N^2a^2}\right)^{\frac{2}{3}} \log\left(38\frac{N^2a^2}{\lambda}\right) - \frac{1}{30}\left(3\lambda Na\right)^{\frac{2}{3}}\right]$ by the sum-rule method and $\omega \approx \sqrt{3}\lambda\big\{1 + (1/6)(\pi/2)^{\frac{2}{3}}[\lambda/(Na)^2]^{\frac{2}{3}} - (1/12)(2/\pi)^{\frac{1}{3}}(\lambda Na)^{\frac{2}{3}}\big\}$ by the time-dependent variational method.

Finally, we present numerical values of the collective-excitation frequency for three sets of parameters that are relevant to recent experiments [10–12]. Our results for parameter sets A and B are close to recent results $(\omega/\omega_z)^2 = 2.85$ and 2.91 in Ref. [8] based on another sum-rule approach and hydrodynamic equations in 1D.

Collective Excitation Frequency

	Sum-rule	Sum-rule (perturbation)	Variational	Variational (perturbation)
Parameter set A	2.89	2.84	2.84	2.83
Parameter set B	2.96	2.94	2.91	2.93
Parameter set C	2.61	1.52	2.56	1.48

Table 1
Our results for the breathing mode collective-excitation frequencies for three sets of parameters relevant to recent experiments. From the left to right, we show results obtained by the sum-rule method [Eq. (1)] with a numerical solution of the GP equation, the lowest order perturbation for the purely 1D results by the sum-rule method, results obtained by the time-dependent variational method, and the lowest order perturbation for the purely 1D results by the time-dependent variational method. Parameter set A: 10^4 ^{23}Na atoms in a harmonic trap with $\omega_\perp = 2\pi \times 360$ Hz and $\omega_z = 2\pi \times 3.5$ Hz, which are relevant to Ref. [10]. Parameter set B: 10^4 ^{7}Li atoms in a harmonic trap with $\omega_\perp = 2\pi \times 4970$ Hz and $\omega_z = 2\pi \times 83$ Hz, which are relevant to Ref. [11]. Parameter set C: 3×10^4 ^{87}Rb atoms in a harmonic trap with $\omega_\perp = 2\pi \times 715$ Hz and $\omega_z = 2\pi \times 14$ Hz, which are relevant to Ref. [12]. (Our perturbational analysis fails for parameter set C.)

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