

# Coreless Vortex in $p$ -wave Superconductor

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## Abstract

We have considered coreless vortices in  $p$ -wave superconductors for arbitrary ratio  $\kappa = \lambda/\xi$  (here  $\lambda$  and  $\xi$  are the magnetic length and the coherence length correspondently). Both order parameter and magnetic field of the coreless vortex decaying from the vortex axis demonstrate essential difference of the vortex structure for different  $\kappa$  parameter and from those calculated for the vortex in liquid  $He_3$ . In particular, strong assymetry usual for some of the components of the order parameter disappears when  $\kappa$  parameter approaches to one.

*Key words:*  $p$ -wave superconductor; flux lines; vortex core

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## 1. Introduction

Order parameter describing Cooper pairs in nonconventional superconductors generally has several components [1]. Examples include description of high  $T_c$  superconductors as a mixture of d-wave and s-wave components [2] and of  $p$ -wave superconductors like heavy fermion  $UPt_3$  [3] or newly discovered  $Sr_2RuO_4$  [4]. The symmetry of the order parameter is related to the crystallographic symmetry group of the material and to the effective attraction mechanism of Cooper pairing. Although the number of charged fields and their transformation properties under rotations are different a common feature of theories describing these diverse systems remains the  $U(1)$  local gauge invariance. It is well known that, while in the simplest one component case the Abrikosov vortices (AV) are the only kind of topological defect, in the multicomponent case other types of defects exist.

In this paper we concentrate on a complex vector field model (describing, in particular, certain superconductors with  $p$ - wave pairing) that possesses an approximate global  $SO(3)$  symmetry. In this case the number of the components of the order parameter

$n = 3$  provided more sophisticated structure of the topological defects. Two topologically distinct "non Abrikosov" (which in this context denote one component topological solitons with additional components vanishing) types had been found: coreless magnetic skyrmions [5] and the "vector vortices" (VV) (pointed out by some of us recently [6]) which have a complex core more typical for superfluid  $He_3$  rather than superconductors. The vector vortices are also not related to unconventional vortices in two-component order parameter models.

We have considered before both structure and dynamical generation of coreless vortices when the ratio  $\kappa = \lambda/\xi$  is rather large,  $\kappa = 10$  (where  $\lambda$  and  $\xi$  are the magnetic penetration depth and the coherence length respectively), while in real  $p$ -wave superconductors it CCC varies in a wide range of  $\kappa > 1/\sqrt{2}$ . The large  $\kappa$  case is simpler to investigate analytically and it enables a qualitative comparison with a very extensively studies case of the tensorial order parameter in superfluid  $He_3$ . In this paper we tackle a more demanding case of intermediate and small  $\kappa$ . We calculated numerically both the structure of the order parameter field and of the magnetic field as function of the distance from the vortex axis and demonstrate essential difference of the vortex structure for different  $\kappa$  parameter.

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## 2. Vector Vortex structure for arbitrary $\kappa$ .

The interaction of the complex vector field theory when the order parameter  $\psi(\mathbf{r}) = (\psi^1, \psi^2, \psi^3)$  is a three component complex vector with magnetic field described by a vector potential  $\mathbf{A}(\mathbf{r})$  is described by the free energy functional

$$F = \int d^2r \left( \frac{K}{2} |(\nabla - i\mathbf{A})\psi^a|^2 - \alpha |\psi^a|^2 + \frac{\beta_1}{2} (\psi^a \psi^{*a})^2 + \frac{\beta_2}{2} |\psi^a \psi^a|^2 + \frac{B^2}{8\pi} \right) \quad (1)$$

Here  $\mathbf{B} = \nabla \times \mathbf{A}$  is the magnetic field,  $a = 1, 2, 3$ . Since the configuration is assumed to be homogeneous in the direction of the magnetic field all the fields are considered in two dimensions,  $\mathbf{r} = (x, y)$ .

We look for a solution in the following  $SO_r(2)$  symmetric ansatz :

$$\psi^1 = c(r) \cos \varphi + id(r) \sin \varphi;$$

$$\psi^2 = -c(r) \sin \varphi + id(r) \cos \varphi;$$

$$\psi^3 = f(r)$$

$$A_x = b(r) \sin \varphi; \quad A_y = -b(r) \cos \varphi$$

Here  $\varphi$  is the azimuthal angle,  $c(r)$ ,  $d(r)$ ,  $f(r)$ ,  $b(r)$  are even real functions of the radial coordinate which have been calculated analitically in the case of large  $\kappa$ .

Substituting the ansatz into Eq.(1), one can obtain the components of the superconducting current:

$$\kappa^2 J_\varphi = \frac{2dc}{r} + F^2 b$$

where  $F^2 = (f^2 + c^2 + d^2)$ . The necessary condition  $J_r = 0$  implies  $A_r = 0$ .

For arbitrary  $\kappa$  magnitude the only numerical solution is possible. We calculated the order parameter  $F^2 = |\psi_1|^2 + |\psi_2|^2 + |\psi_3|^2$ , the single vortex spatial profile of magnetic field, and supercurrent ( $J_\varphi$ , the azimuthal component) for different  $\kappa$ . It shows that the minimal magnitude of the order parameter  $F_{\min}^2 = 0.5$  for  $\kappa = 10$  is monotonically decreased to  $F_{\min}^2 = 0.2$  when  $\kappa = 1$ , before approaching zero at critical  $\kappa_c$ . While it is strongly spatially assymmetric, for large  $\kappa$ , the VV becomes symmetric  $|\psi_1|^2$  component becomes symmetric (Fig.1). In contrast to the order parameter, both the azimuthal component of the superconducting current and magnetic field in the vortex center increase significantly for smal  $\kappa$  parameter values.

The energy  $E(\kappa)$  of the single vortex in p-wave superconductor

$$E(R) = \int_0^{2\pi} d\varphi \int_0^R f\{\psi(\mathbf{r}), \mathbf{A}, \kappa\} r dr$$

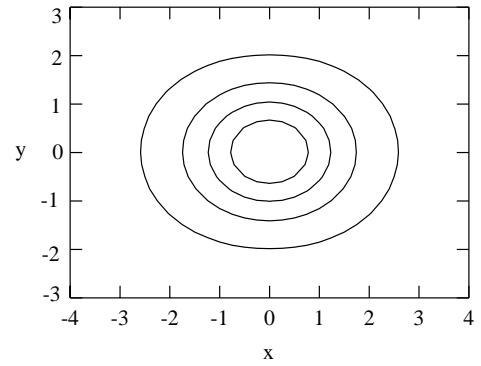


Fig. 1.

(here  $f\{\psi(\mathbf{r}), \mathbf{A}\}$  is the Hamiltonian density). This dependence can be approximately described by the dependence

$$\epsilon = \frac{E(\kappa)}{E(20)} \approx C_1(\ln \kappa + C_2), \quad C_1 = 2.85; \quad C_2 = 2.5$$

The structure of the vortex core might have various static and dynamical implications on the physics of the p-wave superconductors. For example, the existence of a superconducting component in the core must significantly decrease the pinning force and thereby the transport properties.

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