

# Phase separation in magnetically ordered semimetals and half-metals

I. Ya. Korenblit <sup>1</sup>

*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel*

---

## Abstract

We derived a free energy functional, which describes the transition from uniformly ordered magnetic state into a phase-separated or phase-modulated state in semimetals and half-metals. Considering the energy of the slightly inhomogeneous electron gas in the Hohenberg-Kohn approximation, we showed that the scale of the phase-separated regions is fixed by the Coulomb screening of the carriers, and is of order of several lattice constants.

*Key words:* doped magnets; phase separation; EuB<sub>6</sub>; manganites

---

Phase separation (PS) in magnetic systems, with a small concentration,  $n_0$ , of electrons per localized spin (LS), e.g. in slightly doped manganites, has attracted considerable attention recently. (For a review, see [1]).

We consider in this paper the properties of phase-separated systems in the vicinity of the transition point (or line). Close to the transition point the electron gas is only slightly inhomogeneous, i.e. the electron density,  $n(\mathbf{r})$ , has the form  $n(\mathbf{r}) = n_0 + \tilde{n}(\mathbf{r})$ , with  $\tilde{n}(\mathbf{r})/n_0 \ll 1$ , and  $\tilde{n}_{\mathbf{q}=\mathbf{0}} = 0$ , where  $\tilde{n}_{\mathbf{q}}$  is the Fourier transform of  $\tilde{n}(\mathbf{r})$ . Using the Hohenberg-Kohn expression for the free-energy functional (FEF) of a slightly inhomogeneous electron gas [2], we obtain the FEF,  $F[\tilde{n}_{\mathbf{q}}]$ , of the system as follows

$$\begin{aligned} F[\tilde{n}_{\mathbf{q}}] = & \sum_{\mathbf{q}} r_{\mathbf{q}} \tilde{n}_{\mathbf{q}} \tilde{n}_{-\mathbf{q}} \\ & + b \sum_{\mathbf{q}_1 \mathbf{q}_2 \mathbf{q}_3} \tilde{n}_{\mathbf{q}_1} \tilde{n}_{\mathbf{q}_2} \tilde{n}_{\mathbf{q}_3} \delta(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3) \\ & + c \sum_{\mathbf{q}_1 \mathbf{q}_2 \mathbf{q}_3 \mathbf{q}_4} \tilde{n}_{\mathbf{q}_1} \tilde{n}_{\mathbf{q}_2} \tilde{n}_{\mathbf{q}_3} \tilde{n}_{\mathbf{q}_4} \delta(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3 + \mathbf{q}_4). \quad (1) \end{aligned}$$

Here

---

<sup>1</sup> E-mail:korn@ccsg.tau.ac.il

$$\begin{aligned} r(\mathbf{q}) &= \frac{\partial^2 f(n, \eta)}{\partial n^2} + \frac{1}{2\mathcal{N}} \left( \frac{\kappa^2}{q^2} + \frac{q^2}{12k_F^2} \right) \\ &= \frac{\partial^2 f(n, \eta)}{\partial n^2} + \frac{1}{2\mathcal{N}} \left[ \frac{\kappa}{\sqrt{3}k_F} + \frac{\kappa^2}{q^2} \left( 1 - \frac{q^2}{q_0^2} \right)^2 \right], \quad (2) \end{aligned}$$

where  $f(n, \eta)$  includes the free energy of the magnetic system and the electron kinetic energy,  $\eta$  stands for the temperature,  $T$ , magnetic field,  $H$ , and other parameters, specifying the free energy,  $\mathcal{N}$  is the electron density of states at the Fermi energy,  $\kappa^{-1}$  is the Thomas-Fermi screening length,  $\epsilon_F$  and  $k_F$  are the Fermi energy and the Fermi wave vector, and

$$q_0^2 = 2\sqrt{3}k_F\kappa. \quad (3)$$

$b$  and  $c$  in Eq. (1) are constants, which depend on  $\epsilon_F$  and  $n_0$ .

We suppose that the Coulomb interaction is weak, and hence,  $\kappa \ll k_F$ ,  $q_0^2 \ll (2k_F)^2$ .

The Ginzburg-Landau FEF (1) describes a first order liquid-solid-like transition into a periodic structure, with the magnitude of the reciprocal lattice vectors close to  $q_0$ . The transition happens at the the critical line  $\eta_c(n_0)$  given approximately by

$$g(n, \eta) \equiv \frac{1}{2\mathcal{N}} \frac{\partial^2 f(n, \eta)}{\partial n^2} = 0. \quad (4)$$

If the electron spectrum is anisotropic, the inhomogeneous fluctuations at the transition point grow in some special direction. In this case the triad of vectors  $\mathbf{q}_0$  cannot be add to zero, and the third-order term in the functional (1) vanishes. As a result, the transition into the PS state becomes a second order phase transition.

Consider now two systems, which can be described by this approach.

*Europium hexaboride.* This is a ferromagnetic semimetal (FSM), with  $n_0$  of order of  $10^{-3} - 10^{-2}$  [3]. The Eu spins ( $S=7/2$ ) order due to the RKKY indirect coupling. The Fermi energy is much smaller than the s-f exchange energy,  $JS$ , which in his turn is smaller than the band width. The polarized carriers act on the LS as a molecular field, proportional to  $n(\mathbf{r})$ . Hence, the nonuniform fluctuations of  $n(\mathbf{r})$  lead to the fluctuation of the free energy of the LS. The function  $g(n_0, \eta)$  is [4]

$$g(n_0, \eta) = 1 - \frac{3J^2 n_0}{2\epsilon_F T} B' \left( \frac{Jn_0 + g\mu_B H}{T} \right), \quad (5)$$

where  $B(x)$  is the Brillouin function,  $B'(x) = dB(x)/dx$ .

Fig. 1 shows the phase diagram, which follows in this case from Eqs. (4) and (5).

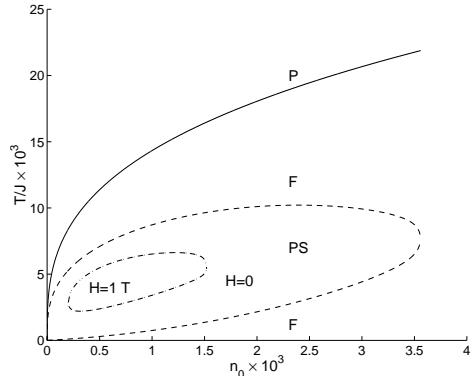


Fig. 1. Phase diagram of a FSM, with  $J = 0.1$  eV,  $S = 3.5$ . The dashed (dashed-dotted) line enclose the phase-separated region, PS, in zero(finite) magnetic field. P and F denote paramagnetic and ferromagnetic phases.

The results account qualitatively for most of the peculiar magnetic properties of  $\text{EuB}_6$  [4]. Specifically, they explain naturally the sequence of transition temperatures observed in this ferromagnet [3,5] as well as the strong effect of a magnetic field on the transition temperature into the PS state [3].

*Slightly doped manganites.* The competition between the antiferromagnetic (AF) exchange,  $I$ , and the double exchange in manganites results in canting of the AF ground state [6]. In the G-type AF the canted state is unstable against phase separation at small  $n$ . The ki-

netic energy of the electrons, stabilizes the canted state, if  $n$  is larger than  $n_c \approx (9/4)^{3/2} (6\pi^2)^{-1} \approx 0.056$  (see Ref. [7] and references therein). The function  $g(n_0, \eta)$  depends only on  $n_0$ , and is proportional to  $n_0^{3/2} - n_c^{3/2}$ . Since the electron spectrum in the manganites is anisotropic, we expect that the phase transition into the PS state is of the second order type. In the PS state a stripe-like structure is formed, with the typical scale of the inhomogeneity,  $r_0 \sim q_0^{-1}$ , determined by the competition between the Coulomb energy, which tends to decrease  $r_0$ , and the gradient energy, which tends to increase it. It is understand here that  $q_0$ , (3), is properly modified to take into account the anisotropy of the spectrum. The scale  $r_0$  is of order of several lattice constants. In the A-type AF the increase of the kinetic energy with  $n$  is weaker than in the G-type AF. The function  $g$  does not depend on  $n_0$ , and is equal to

$$g(t, t') \propto 9.11 \left( \frac{4IS^2 t'}{t^2} \right)^{2/3} - 1, \quad (6)$$

where  $t'(t)$  is the hopping integral across (in) the ferromagnetic layer. The canting happens, if  $t'$  is larger than some critical value. The above theory holds if  $t$  is close to the critical value.

## Acknowledgements

I acknowledge discussions with A. Aharony and O. Entin-Wohlman. This work was supported by the German-Israeli Foundation.

## References

- [1] E. Dagotto, T. Hotta, A. Moreo, Phys. Rep. **344** (2001) 1.
- [2] P. Hohenberg, W. Kohn, Phys. Rev. **136** (1964) B864.
- [3] S. Süllow *et al.*, Phys. Rev. B **57** (1998) 5860.
- [4] I. Ya Korenblit, Phys. Rev. B **64** (2001) 100405(R).
- [5] L. Degiorgi *et al.*, Phys. Rev. Lett. **79** (1997) 14541.
- [6] P. G. de Gennes, Phys. Rev. **82** (1960) 403.
- [7] M. Yu. Kagan, D. I. Khomskii, M. V. Mostovoy, Eur. Phys. J. B **12** (1999) 217.