Mean-field approach to dynamical melting and transverse pinning of moving vortex lattices interacting with periodic pinning

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Abstract

Dynamical melting and transverse pinning of moving vortex lattices in clean superconducting films with periodic pinning are studied by a mean-field treatment of Langevin's equations for the whole vortex lattice, assuming elastic flow. Vortex displacements due thermal fluctuations and to the periodic pinning force are calculated by a perturbation solution of the mean-field equations of motion. The dynamical melting temperature is obtained using Lindemann's criterion. Transverse pinning is demonstrated for motion along the periodic pinning high-symmetry directions and the critical force is estimated.

Key words: dynamical melting; periodic pinning; transverse pinning

The study of dynamical phases and dynamical phase transitions of moving vortices interacting with periodic pinning has received a great deal of attention lately [1]. In recent numerical studies of clean films with periodic pinning arrays [2,3] and of periodic Josephson Junction Arrays [4] two dynamical phenomena are reported: dynamical melting of a moving vortex lattice (VL) into a moving liquid, and transverse pinning. The dynamical melting temperature is found to approach the equilibrium one in the limit of very large center of mass (CM) velocity, and to decrease as the CM velocity decreases. Transverse pinning occurs when the CM velocity is oriented along one the high-symmetry directions of the periodic pinning potential, and remains pinned to this direction under a transverse driving force less than a critical value. In this paper we propose a simple analytical model for the moving VL dynamics, and use it to estimate the dynamical melting temperature and the transverse critical force.

We consider a two-dimensional VL at temperature T and with N_v vortices, interacting with a periodic pinning array and moving with CM velocity $\mathbf{v}$. We assume elastic motion, so that vortex-vortex interactions can be approximated by harmonic forces. Our model considers a single vortex, and approximates the elastic force exerted on it by the rest of the lattice by that of a spring with constant κ. This is equivalent to a mean-field treatment of Langevin's equations for the full elastic problem [5]. The equation of motion for the vortex in the frame moving with the CM velocity is then,

$$\eta\frac{d\mathbf{u}_l}{dt} = -\kappa(\mathbf{u}_l - \bar{\mathbf{u}}) + \mathbf{F}^{v-p}(\mathbf{R}_l + \mathbf{u}_l + \mathbf{v}t) \\ \mathbf{f}_d - \eta\mathbf{v} + \Gamma_j\,, \qquad (1)$$

where η is the friction coefficient, $\mathbf{R}_j$ and $\mathbf{u}_j$ denote, respectively, the VL positions and displacements in the CM frame, $\bar{\mathbf{u}} = \frac{1}{N_v}\sum_j \mathbf{u}_j$ is the (time-independent) displacement of the CM, $\mathbf{F}^{v-p}(\mathbf{r})$ is the pinning force, $\mathbf{f}_d$ is the driving force, and Γ_j is the random force appropriate for temperature T. By definition, and according to Eq. (1), the CM velocity must satisfy $\eta\mathbf{v} = \mathbf{f}_d + \frac{1}{N_v}\sum_{j=1}^{N_v}\mathbf{F}^{v-p}(\mathbf{R}_l + \mathbf{u}_l + \mathbf{v}t)$. Since, by assumption, $\mathbf{v}$ is independent of time, $\mathbf{f}_d$ must depend on time as well as on the random force. In order to obtain physical results, this equation is averaged over time and over the random force distribution, that is

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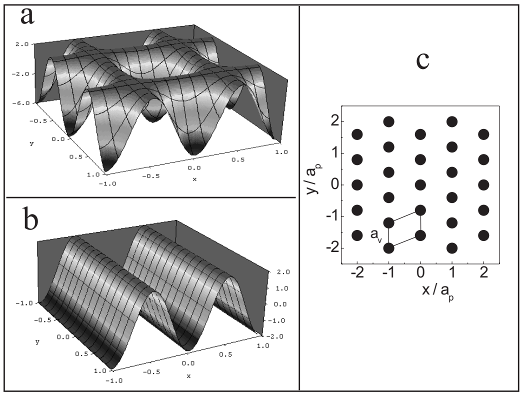

Fig. 1. (a) Periodic pinning potential for $\Phi_2 = \Phi_1/2$ in units of Φ_1 (b) Average of (a) in [0,1] direction. In (a) and (b) x and y in units of a_p. c) Vortex lattice commensurate with washboard shown in (b).

$\eta\mathbf{v} = \mathbf{F}_d + \frac{1}{N_v}\sum_{i=1}^{N_v}\frac{1}{\tau}\int_0^\tau \left\langle \mathbf{F}^{v-p}\left(\mathbf{R}_l + \mathbf{u}_l + \mathbf{v}t\right)\right\rangle$, where $\mathbf{F}_d$ is interpreted as the force due to the applied current, $\langle\rangle$ denotes average over the random force distribution, and τ is a time large compared with the characteristic times in Eq. (1). This equation and Eq. (1) define our model. They can be solved by perturbation theory, expanding $\mathbf{F}^{v-p}$ in powers of $(\mathbf{u}_j - \bar{\mathbf{u}})$, which is small for large v[6]. Here we consider only the lowest order, which corresponds to setting $\mathbf{u}_j = \bar{\mathbf{u}}$ in $\mathbf{F}_j^{v-p}$.

We assume that the pinning array is square, with lattice parameter a_p, and that the pinning potential is $\Phi^{v-p}(\mathbf{r}) = \Phi_1[\cos(Qx) + \cos(Qy)] + \Phi_2[\cos(Q(x+y)) + \cos(Q(x-y))]$, where $Q = \frac{2\pi}{a_p}$ and x (y) is along the [1,0] ([0,1]) direction. The pinning potential for $\Phi_2 = \Phi_1/2$ is shown in Fig. 1a). The pinning force is given by $\mathbf{F}^{v-p}(\mathbf{r}) = -\nabla\Phi^{v-p}(\mathbf{r})$

The moving VL structure depends on the direction of motion. To determine it we consider the limit $v \to \infty$, where the moving vortices feel only the pinning potential averaged in the direction of motion, and the dynamical phase in the CM frame reduces to the equilibrium one [2]. The average of the above described pinning potential along each one of [1,0], [0,1], [1,1] and [-1,1] is a washboard, periodic in the perpendicular direction, such as the one shown in Fig. 1.(b). The corresponding dynamical phase at low T is a VL commensurate or incommensurate with the corresponding washboard, depending on the vortex density. For other directions of motion, the average potential is constant, and the VL is triangular. Transverse pinning occurs only in the commensurate VL .

First we consider dynamical melting of triangular lattices. To obtain the dynamical melting temperature, $T_{dm}(\mathbf{v})$, we use Lindemann's criterion as $u^2 = \frac{1}{N_v}\sum_{j=1}^{N_v}\frac{1}{\tau}\int_0^\tau \left\langle \mid \mathbf{u}_j(t) \mid^2 \right\rangle = c_L^2 a_v^2$, where a_v is the VL lattice parameter. Solving Eq. (1) for $\mathbf{u}_j(t)$ in the lowest order approximation (in this case $\bar{\mathbf{u}} = 0$) we obtain $\mathbf{v} = \mathbf{F}_d/\eta$ and $k_B T_{dm}(\mathbf{v}) = \frac{\kappa a_v^2 c_L^2}{2} - \frac{\kappa}{2}\sum_{\mathbf{Q}}\frac{Q^2|U_{\mathbf{Q}}|^2}{\eta^2(\mathbf{Q}\cdot\mathbf{v})^2+\kappa^2}$. In the $v \to \infty$ limit $T_{dm} \to \frac{\kappa a_v^2 c_L^2}{2}$, which coincides with the equilibrium melting temperature for the VL, obtained by the dislocation unbiding theory, if $\kappa c_L^2 = \phi_0^2/(16\pi^3\sqrt{3}\Lambda a_v^2)$, where Λ is the film effective penetration depth [7]. For finite v, T_{dm} decreases as v decreases. More details of this calculation are given in Ref.[5]

Next we discuss tranverse pinning of a commensurate VL moving along [0,1]. We assume that the VL has the structure shown in Fig. 1.(c), which is commensurate with the pinning array along [1,0]. This VL is similar to the ones found in numerical simulations [2]. Transverse pinning follows from the $\mathbf{v}$ vs. $\mathbf{F}_d$ equation, which in the lowest order approximation becomes, $\eta\mathbf{v} = \mathbf{F}_d + \Phi_1 Q\sin(Q\bar{u}_x)\hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$ is the unit vector along [1,0]. The solutions of this equation are $v_y = F_{dy}/\eta$ and, since $v_x = 0$ by assumption, $\bar{u}_x = -Q^{-1}\arcsin(F_{dx}/\Phi_1 Q)$. For $F_{dx} > \Phi_1 Q$ there is no solution for $\bar{u}_x$, which means that critical transverse force is $F_c = 2\pi\Phi_1/a_p$. This is just the force along x needed to depin the commensurate VL from the washboard at $T = 0$.

In conclusion then, we introduce a simple dynamical model that is capable of predicting dynamical melting temperatures in qualitative agreement with numerical simulation ones, and that allows calculation of the critical transverse pinning force. Here we consider only to the lowest order in perturbation, but the calculations can be extended to higher orders.

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