# Bose-Fermi mixed condensates of atomic gas with Boson-Fermion quasi-bound state

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**Abstract**

The phase structures of the boson-fermion (B and F) mixed condensates of atomic gas are discussed under the existence of boson-fermion composite fermions (quasi-bound states) BF from the equilibrium in $\mathrm{B} + \mathrm{F} \leftrightarrow \mathrm{BF}$. Especially we discuss the competitions between the BF degenerate states and the Bose-Einstein condensates (BEC) in low-T. The criterion for the BEC realization is obtained from the algebraically-derived phase diagrams at $T = 0$.

*Key words:* Bose-Einstein condensate ; Fermi degenerate gas ; Quantum atomic gas ;

## 1. Introduction

Experimental successes of the BEC and the fermi degenerate systems of the trapped atomic gas have opened up renewed interests in the boson-fermion mixed condensates, which are expected to show many interesting physical phenomena [1].

In case that the boson-fermion interaction is enough attractive, the boson-fermion pairs can make quasi-bound states, which behave as composite fermions BF, and produce new phases as the BF degenerate state.

In this paper, we discuss the phase structures of the mixed condensates under the existence of the quasi-bound states with solving the equilibrium condition for the reaction: $\mathrm{B} + \mathrm{F} \leftrightarrow \mathrm{BF}$. Especially interesting is a competition between the BF degenerate states and the BEC of unpaired bosons; the energy-reduction in the BF binding energies v.s. that in the boson kinetic energies in the BEC. If the BF binding energy is enough large, the BF pairs can exhaust the bosons and the BEC will not appear in the mixed condensates.

## 2. Equilibrium condition

Let's consider the uniform system of porallized bosons and fermions (B and F), with the masses $m_{\mathrm{B}}$ and $m_{\mathrm{F}}$. The total numbers of B and F should be conserved and their densities are $n_{\mathrm{Btot}}$ and $n_{\mathrm{Ftot}}$ each other. A quasi-bound state (composite fermion) BF is assumed to exist with the mass $m_{\mathrm{BF}}$.

In states with temperature $T$, because of the equilibrium $B + F \leftrightarrow BF$, a part of the atom B and F are paired in the BF states, and the others are in free unpaired states. The equilibrium condition is given by

$$\mu_{\mathrm{B}} + \mu_{\mathrm{F}} = \mu_{\mathrm{BF}} + \Delta m c^2, \tag{1}$$

where $\mu_{\mathrm{B,F,BF}}$ are chemical potentials of atoms B, F, BF each other, and $\Delta m = m_{\mathrm{BF}} - m_{\mathrm{B}} - m_{\mathrm{F}}$ is a binding energy of the BF state. The chemical potentials in (1) are obtained by the density formulae of the free bose/fermi gas:

$$n_{\mathrm{B}} = \frac{(m_{\mathrm{B}})^{3/2}}{\sqrt{2}\pi^2} \int_0^\infty \frac{\sqrt{\epsilon} d\epsilon}{e^{(\epsilon - \mu_{\mathrm{B}})/k_B T} - 1}, \tag{2}$$

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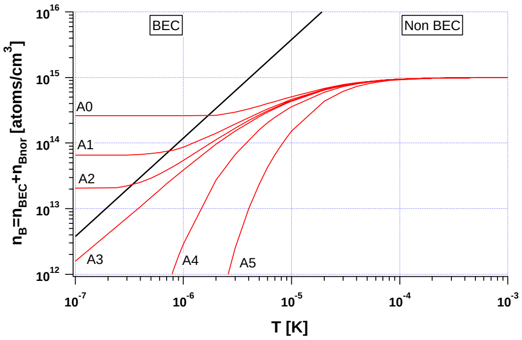


Fig. 1. $T$-dependence of boson density

$$n_a = \frac{(m_a)^{3/2}}{\sqrt{2}\pi^2}\int_0^\infty \frac{\sqrt{\epsilon}d\epsilon}{e^{(\epsilon-\mu_a)/k_BT}+1}, \quad (a = \mathrm{F}, \mathrm{BF}) \quad (3)$$

where $k_B$ is a Boltzmann constant, and $n_{\mathrm{B,F,BF}}$ are the densities of the free (unpaired) B and F and the composite BF.

Solving eq. (1) with (2-3) under the atom number conservation for B and F: $n_{\mathrm{B}} + n_{\mathrm{BF}} = n_{\mathrm{Btot}}$ and $n_{\mathrm{F}} + n_{\mathrm{BF}} = n_{\mathrm{Ftot}}$, we obtain the densities $n_{\mathrm{B,F,BF}}$ as functions of $T$ and $n_{\mathrm{Btot,Ftot}}$.

When $T$ and $n_{\mathrm{B}}$ satisfy $T < T_C \equiv \frac{2\pi\hbar^2}{m_B k_B}\left(\frac{n_B}{2.613}\right)^{2/3}$, a part of free bosons condensates into the BEC, and $\mu_{\mathrm{B}}$ becomes zero. In that case, the equilibrium condition becomes $\mu_{\mathrm{F}} = \mu_{\mathrm{BF}} + \Delta mc^2$. When the BEC exists, The condensed- and normal-component densities of bosons $n_{\mathrm{BEC,Bnor}}$ are defined by $n_{\mathrm{BEC}} = n_{\mathrm{B}}\left[1 - \left(\frac{T}{T_C}\right)^{3/2}\right]$, and $n_{\mathrm{Bnor}} = n_{\mathrm{B}} - n_{\mathrm{BEC}}$.

## 3. Results and summary

As an typical example, we show the $T$-dependence of free boson density $n_{\mathrm{B}} = n_{\mathrm{BEC}} + n_{\mathrm{Bnor}}$ when $n_{\mathrm{Btot}} = n_{\mathrm{Ftot}} = 10^{15}\,\mathrm{atoms/cm^3}$ in Fig. 1. The lines A0-A5 are for $\Delta m = (0, -3, -4, -4.71, -10) \times 10^{-6}\,\mathrm{K}$. The oblique straight line is the critical border of the BEC region. The $n_{\mathrm{B}}$ are found to decrease with decreasing $T$; it is because the number of composite fermions increases in low-$T$. In small $\Delta m$ cases (A0-A3), the $n_{\mathrm{B}}$ is still large and free bosons can condensate into the BEC in low-$T$, but, in large $\Delta m$ cases (A4,A5), free bosons are exhausted in making composite fermions and the $n_{\mathrm{B}}$ becomes too small for the BEC realization. The line A6 corresponds to the critical case. In high-$T$ region, all composite fermions dissociate into free bosons and fermions, so that $n_{\mathrm{B}}$ approaches to $n_{\mathrm{Btop}}$. In Fig. 2, the variations of $T_C$ for the BEC transitions are shown in $n_{\mathrm{F}}/n_{\mathrm{B}}$ =0.3 (A), 0.5 (B), 0.8 (C), 1 (D), 1.2 (E), 1.36 (F), 2 (G).

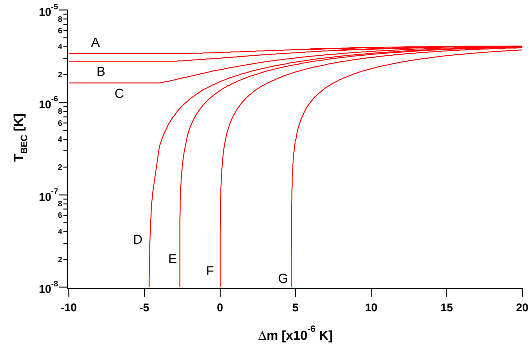


Fig. 2. $T_C$ for BEC transition v.s. $\Delta m$

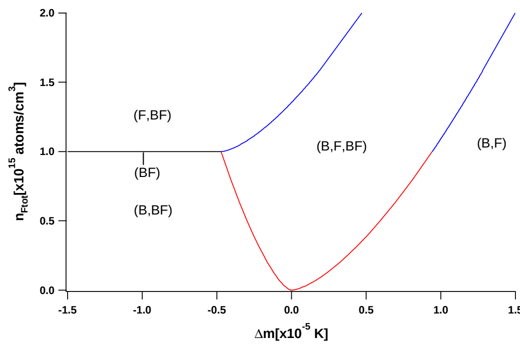


Fig. 3. Phase diagram at $T = 0\,\mathrm{K}$ ($n_{\mathrm{Btot}} = 10^{15}\,\mathrm{atoms/cm^3}$)

When $n_{\mathrm{Btot}} < n_{\mathrm{Ftot}}$, the $n_{\mathrm{B}}$ has similar $T$-dependence as in Fig. 1. When $n_{\mathrm{Btot}} > n_{\mathrm{Ftot}}$, the BEC always occur in enough low-$T$ because, after all fermions are paired, the free bosons still remain.

At $T = 0$, the condition (1) becomes $0 + \epsilon_{\mathrm{F}} = \epsilon_{\mathrm{BF}} = \Delta mc^2$, where $\epsilon_a = \frac{(3\pi^2)^{2/3}}{2^{1/3}m_a} n_a^{2/3}$ $(a = B, BF)$. It can be solved algebraically and gives the phase structures at $T = 0$. In Fig. 3, we show the phase diagrams in $n_{\mathrm{Ftot}} - \Delta m$ plane when $n_{\mathrm{Ftot}} = 10^{15}\,\mathrm{atoms/cm^3}$, where the symbol (B,F,BF) means the coexistence of free bosons and free and composite fermions, and so on.

From this diagram, we can read off the criterion for the BEC to occur; it should occur in the regions when free bosons exist at $T = 0$, e.g the ones with the symbol B in Fig. 2.

In summary, we studied the role of the composite fermion in the boson-fermion mixed condensates and its phase structure in low-$T$. The more details and further applications of the present results should be discussed in further publication[2].

## References

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