

# Electron dynamics in quantum wells under tilted magnetic field and intense AC field

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## Abstract

The electron dynamics in a double quantum well under strong AC electric field and tilted magnetic field is studied using a non-perturbative Floquet approach. For  $B_{\parallel} = 0$ , the quasienergy spectra show two types of crossings: those related to different Landau levels (LL), and those due to dynamic localization (DL). We find that the system exhibits DL at the same values of the AC field suggesting a hidden dynamical symmetry identified with different parity operations. We analyze also the time evolution of the system, monitoring the elapsed time to return to a given well, and find a complex behavior which is studied in detail.

*Key words:* dynamic localization; tilted magnetic field; double quantum well

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The development of free-electron lasers [1], which can be continuously tuned in the terahertz (THz) range, have made possible the experimental study of intense AC field effects in semiconductor heterostructures. Phenomena studied include the coherent suppression of tunnelling or *dynamic localization* (DL) between coupled wells, the collapse of minibands in superlattices, the appearance of absolute negative conductance, and the AC Stark effect [2]. Strong AC fields on semiconductors in the presence of magnetic fields either parallel ( $B_{\parallel}$ ), or perpendicular ( $B_{\perp}$ ), to the interfaces, are also of interest, as the spectra and tunnelling amplitudes are quite sensitive to fields. In fact, quantum wells in *tilted* magnetic fields have been used as excellent experimental probes of the transition to quantum chaos in a mesoscopic system [3].

Here we address the problem of a double quantum well (DQW) *simultaneously* in *tilted* magnetic and strong AC fields, in order to explore the role of symmetry breaking on coherent tunnelling phenomena. We utilize a *non-perturbative* approach based on a Floquet-Fourier formalism that describes the periodic

field problem in terms of a set of quasienergy levels and their harmonics.

The Hamiltonian for an electron in a double quantum well structure under a tilted magnetic field and a strong AC field can be written as

$$H = H_0 + eFz \cos(\omega t), \quad (1)$$

where  $H_0 = (\mathbf{P} - e\mathbf{A})^2/2m^* + V_0$  is the Hamiltonian for the non-driven system. We have chosen the growth direction along the  $z$ -axis and the direction of the magnetic field in the  $x$ - $z$  plane  $\mathbf{B} = (B_{\parallel}, 0, B_{\perp})$ .

Following the procedure developed by Shirley [4], we solve the time-dependent Schrödinger equation by expanding the Floquet states in a Fourier series for the temporal component, and linear combinations of the  $B_{\parallel} = 0$  solutions of  $H_0$  for the spatial component (harmonic oscillator  $|n\rangle$  in plane with the symmetrical and antisymmetrical  $|s\rangle$  solutions of the DQW). This leads to the time-independent infinite matrix eigenvalue equation

$$\sum_{s,n,m} \left\{ \left[ E_s + \hbar\omega_{\perp}(n + \frac{1}{2}) - \varepsilon + m\hbar\omega \right] \delta_{s,s'} \delta_{n,n'} \delta_{m,m'} \right\}$$

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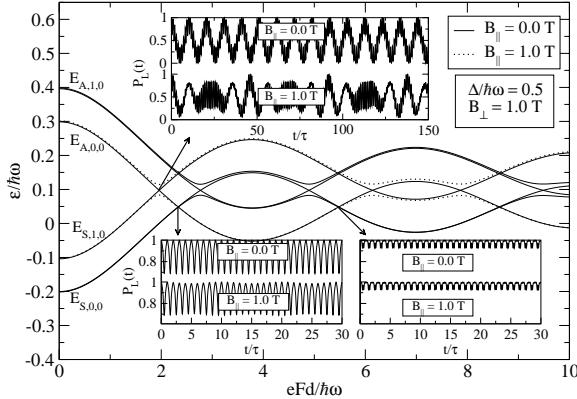


Fig. 1. First Brillouin zone of quasienergies (in units of  $\hbar\omega$ ) as a function of  $eFd/\hbar\omega$  (intensity of AC field), for  $\Delta/\hbar\omega = 0.5$  and  $B_\perp = 1.0$  T. Solid ( $B_\parallel = 0$ ) and dotted ( $B_\parallel = 1.0$  T) lines show the quasienergy dependence with in-plane magnetic field. The label  $E_{s,n,m}$  refers to the eigenvalues of the system for vanishing  $F$  and  $B_\parallel$  ( $s$  indicates symmetrical or antisymmetrical state;  $n$  is the LL index; and  $m$  indicates the quasienergy harmonic index). The insets show the time evolution  $P_L(t)$  of the system initially in the left well for the points indicated by arrows.

$$+ \left[ \frac{e^2 B_\parallel^2}{2m^*} z_{s,s'}^2 \delta_{n,n'} - \frac{e^2 B_\parallel B_\perp}{m^*} z_{s,s'} x'_{n,n'} \right] \delta_{m,m'} \\ + \frac{eFz_{s,s'}}{2} (\delta_{m,m'-1} + \delta_{m,m'+1}) \delta_{n,n'} \} C_{s,n,m} = 0, \quad (2)$$

where  $z_{s,s'}^2 = \langle s|z^2|s' \rangle$ ,  $z_{s,s'} = \langle s|z|s' \rangle$ ,  $x'_{n,n'} = \langle n|x'|n' \rangle$ , and  $E_s = \pm\Delta/2$ , with  $\Delta$  being the DQW splitting due to the tunnelling.

We make a systematic study of the system exploring and comparing both quasienergy spectra and the time evolution of the system. In Fig. 1 we show the first Brillouin zone of quasienergies (in units of  $\hbar\omega$ ) as a function of  $eFd/\hbar\omega$  (intensity of AC field), for  $\Delta/\hbar\omega = 0.5$  and  $B_\perp = 1.0$  T. We find that in the absence of  $B_\parallel$ , the problem can be understood in terms of independent sets of two state systems for different LL (solid line). When  $B_\parallel$  is turned on, however, level crossings allowed for  $B_\parallel = 0$ , develop into anticrossings and level mixing (dotted line), reflecting the symmetry breaking effects of the tilted field. This effect is accompanied by a complex time evolution of the system (see upper inset in Fig. 1 for  $B_\parallel = 1.0$  T and stroboscopic map in Fig. 2(c)), and is closely associated with the onset of chaos [3]. On the other hand, the inclusion of  $B_\parallel$ , produces no effect in the crossing point  $eFd/\hbar\omega \simeq 2.33$ , as we can see in the lower left inset in Fig. 1. At this point we have DL, since the probability for the system to stay in the left well is very high and never goes to zero. It is important to note that the parallel magnetic field has no effect on the position of this DL point. For the crossing point at  $eFd/\hbar\omega \simeq 5.48$  the lower right inset shows similar results, except for the fact that the time

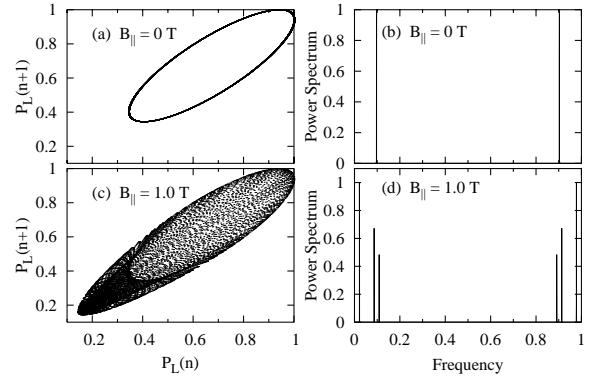


Fig. 2. Stroboscopic map of the probability of staying in the left well in multiples of the period of the AC field ( $n\tau$ ) with  $\Delta/\hbar\omega = 0.5$ ,  $B_\perp = 1.0$  T, and  $eFd/\hbar\omega = 1.96$  (crossing/anticrossing point) for (a)  $B_\parallel = 0$  T, (c)  $B_\parallel = 1.0$  T, and respective, (b) and (d), power spectra of the time evolution sequence (Frequency is in units of  $\tau$ ).

evolution presents better dynamic localization (notice the minima in  $P_L \simeq 0.92$ ).

In Fig. 2(a) we show the stroboscopic map of the probability of staying in the left well (referent to the upper inset in Fig. 1) in multiples of the period of the AC field ( $n\tau$ ) for  $B_\parallel = 0$  T. Its respective power spectrum is shown in Fig. 2(b). The system is completely periodic with frequency  $\simeq 0.1$  that is associated with the level separation  $E_{S,0,0}$  and  $E_{S,1,0}$ . Only transitions with the same LL are allowed. Figure 2(c) shows the result for  $B_\parallel = 1.0$  T with its power spectrum in Fig. 2(d). The time evolution now presents a complex behavior that comes from the levels repulsion at the anticrossing points associated with incommensurate frequency ratios.

These theoretical studies provide a unifying scheme to understand experiments in THz fields [1], and yield interesting effects as function of the structure parameters.

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## References

- [1] S. J. Allen, Nucl. Instrum. Meth. B **144** (1998) 130.
- [2] M. Grifoni, P. Hänggi, Phys. Rep. **304** (1998) 229.
- [3] T. M. Fromhold, A. A. Krokhin, C. R. Tench, S. Bujkiewicz, P. B. Wilkinson, F. W. Sheard, L. Eaves, Phys. Rev. Lett. **87** (2001) 046803.
- [4] Jon H. Shirley, Phys. Rev. **138** (1965) B979.