Quasiclassical Theory of Superconducting Multi-Layers

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Abstract

We study the density of states of a thin normal metal layer in proximity to a bulk superconductor in the clean as well as in the dirty limit. We find that for increasing impurity scattering a mini-gap is established in the normal metal. Furthermore we consider a spin-active surface of the normal metal (e.g. due to an additional ferromagnetic layer) and observe a suppression of the mini-gap.

Key words: superconductivity; quasiclassical theory; multi-layers

1. Introduction

We study a system consisting of a thin normal metal layer on top of a bulk superconductor as shown in Fig. 1 ($d_n \approx \xi_0$, ξ_0: superconducting coherence length). We calculate the density of states (DOS) in the normal metal layer using the quasiclassical theory of superconductivity. We generalize the earlier work of Belzig et al. [1] by considering arbitrary impurity concentrations and reduced transparency between the normal metal and the superconductor. Furthermore we examine a spin-active surface which can be realized by an insulating ferromagnetic layer when in contact with the normal metal.

2. Quasiclassical Theory

In thermal equilibrium the quasiclassical Green's function $\hat{g}(E, \mathbf{p}_F; \mathbf{r})$ is determined by the Eilenberger equation and the normalization condition:

$$[\hat{\tau}_3 E + i\hat{\Delta}(\mathbf{p}_F, \mathbf{r}) + i\hat{\sigma}(\mathbf{r}), \hat{g}] + i\mathbf{v}_F \cdot \nabla \hat{g} = 0, \tag{1}$$

$$\hat{g}^2 = \hat{1}. \tag{2}$$

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The hats denote the 4×4 matrix structure, which combines spin and particle-hole space ($\hat{\tau}_i / \hat{\sigma}_i$ are the Pauli matrices in particle-hole/spin space); for simplicity we assume an identical spherical Fermi surface in the normal metal and the superconductor. The s-wave order parameter $\hat{\Delta}$ and the impurity self-energy $\hat{\sigma}$ (in Born approximation) must be determined self-consistently:

$$\hat{\Delta}(\mathbf{r}) = i\hat{\sigma}_2 \pi T \mathcal{N}_0 V \sum_{|E_n| < E_c} \frac{1}{2} \mathrm{Tr}_\sigma [i\hat{\sigma}_2 \hat{g}_s(iE_n; \mathbf{r})], \tag{3}$$

$$\hat{\sigma}(E; \mathbf{r}) = \frac{1}{2\tau} \hat{g}_s(E; \mathbf{r}). \tag{4}$$

Here $\mathcal{N}_0$ is the DOS of the superconductor in the normal state, V is the pairing interaction, and $\hat{g}_s$ is the s-wave part of the Green's function; Tr_σ is the trace in spin space. This approach allows us to study the crossover from the clean ($1/2\tau = 0$) to the dirty ($1/2\tau > T_c$) limit.

The quasiclassical theory is not directly applicable at interfaces and the Eilenberger equation must be sup-

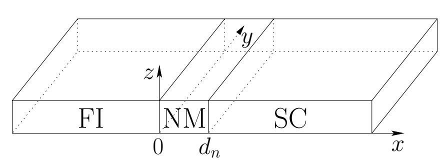

Fig. 1. The F-N-S-structure studied in this work.

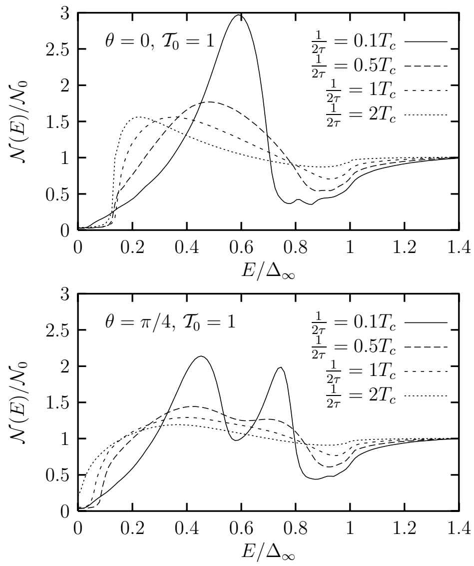

Fig. 2. For ideal transparency and large $1/2\tau$ the mini-gap, which is present for $\theta = 0$, is suppressed for $\theta = \pi/4$ with increasing impurity scattering; for small $1/2\tau$ the splitting of the two spin-channels for $\theta = \pi/4$ can be observed.

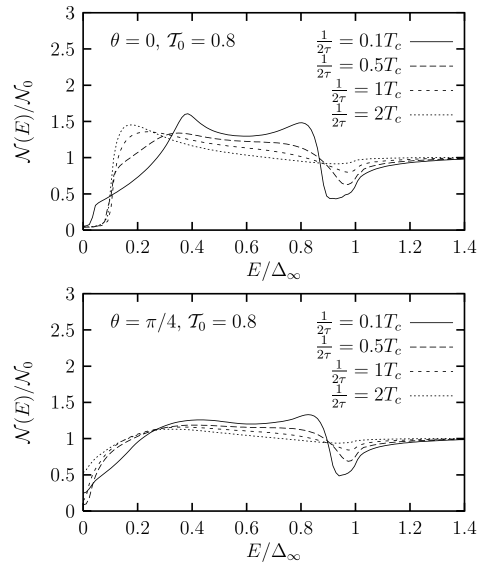

Fig. 3. For reduced transparency the mini-gap is also suppressed in the magnetic case with increasing impurity scattering.

plemented by appropriate boundary conditions. The NS-interface characterized by the transparency $\mathcal{T}_0$ is incorporated with the help of Zaitsev's boundary conditions [2]. The spin-flip processes at the ferromagnetic layer are taken into account by the boundary conditions of Tokuyasu et al. [3] where a phenomenological parameter θ describes the degree of spin-rotation. In this paper we use the boundary condition in terms of the Maki-Schopohl parameterization of the Green's function [4,5].

3. Results

We calculated the DOS in the normal metal at $x = 0$ for $d_n = 1.1\xi_0$ for ideal ($\mathcal{T}_0 = 1$, see Fig. 2) and reduced transparency ($\mathcal{T}_0 = 0.8$, see Fig. 3) of the NS-contact. We compared the case of a spin-active surface ($\theta = \pi/4$) with the spin-conserving case ($\theta = 0$) for various impurity concentrations. The order parameter was determined self-consistently at $T = 0.1T_c$. In the dirty limit with a spin-conserving surface a mini-gap can be observed; the DOS is almost unaffected when varying the transparency. Otherwise in the clean limit a clear dependence on the transparency occurs. Note that at a spin-active surface the mini-gap is suppressed.

The sub-gap structure of the DOS in the normal metal stems from bound states due to Andreev reflection at the superconductor. In the clean case their energies can be calculated for a constant order parameter in the superconductor and ideal transparency at the NS-contact with the result

$$E^2 = \frac{|\Delta|^2}{2}\left[1 + \cos\theta\cos\frac{2EL}{v_F} \pm \sin\theta\sin\frac{2EL}{v_F}\right] \qquad (5)$$

with $L = 2d_n p_F/p_{F,x}$; the signs $\pm$ describe the bound states in the two spin channels. In the non-magnetic case the low energy contribution to the DOS comes from trajectories with small angles of incidence ($L \gg d_n$). These states are strongly affected by impurities in the normal metal. Their spectral weight is shifted to higher energies which results in a mini-gap. For a magnetic surface the energies of these states are shifted up or down depending on the spin channel; this means that the spectral weight at low energies increases, and hence the mini-gap is suppressed.

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