Local density of states of normal-superconducting proximity contact systems with arbitrary concentration of impurities

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Abstract

The superconducting order induced in the normal layer by the proximity effect has considerable influence on the density of states of the normal layer. Belzig et al. reported that, when the normal layer is thin, a mini-gap in the density of states appears in the dirty limit. We study the local density of states of normal-superconducting proximity contact systems with arbitrary concentration of impurities by quasi-classical Green's function method. It is found that the local density of states has also a mini-gap at intermediate concentrations of impurities. The mean free path dependence of the mini-gap is reported.

Key words: density of states; proximity effect; impurity effect; quasi-classical Green's function

The density of states (DOS) of normal metal layer in contact with superconductor has attracted a lot of attention. In the clean limit, DOS in the normal layer is strongly suppressed at low energies ($\varepsilon \ll \Delta$), but there is no energy gap, as shown by de Gennes and Saint-James.[1] On the other hand, Belzig et al.[2] recently found using the Usadel equation that in a dirty thin normal layer a mini-gap appears in the density of states and its magnitude is proportional to the inverse square of the layer width.

In this paper, we study the local DOS of normal-superconducting(N-S) proximity contact systems with arbitrary concentration of impurities by the quasi-classical Green's function method. We show that the local density of state has also a mini-gap at intermediate concentrations of impurities and that, when the normal layer width L_N is fixed, the mini-gap has a maximum around $l \simeq L_N$ as a function of the mean free path l in the N-side.

We consider the quasi-classical Green's function in an N-S proximity contact system. In our model, the N-side is a normal metal with the width L_N and the impurity effect is characterized by the mean free path l. The S-side is a semi-infinite and clean superconducting metal. We assume that the boundary at $z = -L_N$ is completely specular. The N-S interface at $z = 0$ is also assumed to have translational symmetry in the x-y plane and has the reflection coefficient R. According to Ashida et al.,[3] the quasi-classical Green's function which satisfies the boundary conditions at the N-S interface and also at the layer ends can be obtained by using the evolution operators. For instance, in the N-layer,

$$g_{\alpha\alpha}(z) = -i\Big(A_\alpha(z) - \frac{1}{2}\mathrm{tr}A_\alpha(z)\Big)/D \tag{1}$$

$$A_\alpha(z) = -U^N_\alpha(z,-L_N)U^N_+(-L_N,0)(P_S + R\bar{P}_S) \times U^N_-(0,-L_N)U^N_\alpha(-L_N,z) \tag{2}$$

$$D = \sqrt{\Big(\frac{1}{2}\mathrm{tr}A_\alpha\Big)^2 - \det A_\alpha} \tag{3}$$

$$P_S = \lim_{L_S\to\infty} \frac{U^S_+(0,L_S)U^S_-(L_S,0)}{\mathrm{tr}\Big(U^S_+(0,L_S)U^S_-(0,L_S)\Big)}, \tag{4}$$

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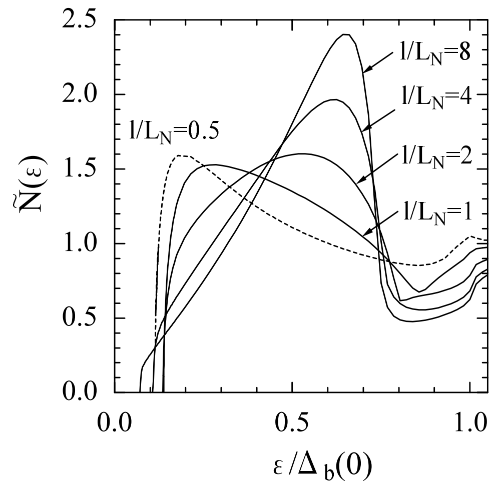

Fig. 1. Local density of states at the surface of the N-layer in the case $\tilde{L}_N = 1$ for several mean free paths.

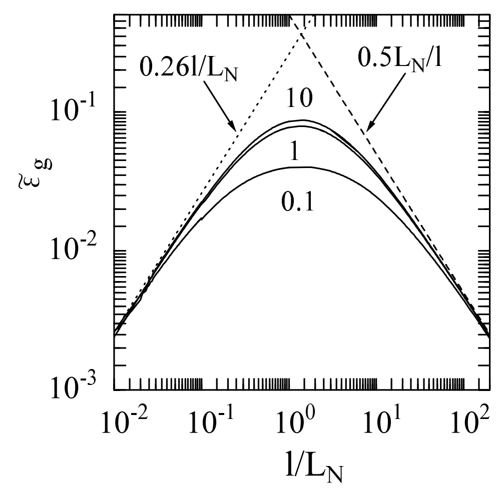

Fig. 2. l dependence of $\tilde{\varepsilon}_g$ in the cases $\tilde{L}_N = 0.1, 1, 10$. The dotted line denotes $\tilde{\varepsilon}_g = 0.26l/L_N$, and the dashed line denotes $\tilde{\varepsilon}_g = 0.5L_N/l$.

where $\bar{P}_S$ is the cofactor matrix of P_S. In this paper, we treat the impurity scattering effects within the Born approximation. Therefore, the evolution operator in each layer obeys the following equation:[3]

$$\partial_z U_\pm(z,z') = \pm \frac{i}{v_{Fz}} \begin{pmatrix} \tilde{\varepsilon}(z) & \tilde{\Delta}(z) \\ -\tilde{\Delta}(z) & -\tilde{\varepsilon}(z) \end{pmatrix} U_\pm(z,z'), \quad (5)$$

$$U_\pm(z,z) = 1, \quad (6)$$

where $v_{Fz} = v_F \cos\theta$ is the z-component of the Fermi velocity and

$$\tilde{\varepsilon}(z) = \varepsilon + i < g(z) > /2\tau, \quad (7)$$

$$\tilde{\Delta}(z) = \Delta(z) + < f(z) > /2\tau. \quad (8)$$

Here $\Delta(z)$ is the pair potential, τ is the elastic scattering time due to impurities and $< g(z) >$ and $< f(z) >$ are the angle average of the diagonal and off-diagonal element of the quasi-classical Green's function, respectively. Using the above quasi-classical Green's function, the local density of states $N(\varepsilon, z)$ in the N-side is given by

$$N(\varepsilon,z) = \frac{N(0)}{2} \Im < \mathrm{tr}\rho_3 g_{++}(\varepsilon + i0, z) >, \quad (9)$$

where $N(0)$ is the density of state at the Fermi energy in the normal metal and the bracket means the angle average $< \cdots > = \int_0^\pi d\theta \sin\theta(\cdots)$.

We have solved numerically the set of self-consistent equations from (1) to (8). In the numerical calculations, we have further assumed that the S-side has a constant pair potential $\Delta_b(0)(= 0.56\pi T_c)$ and that the reflection coefficient of the N-S interface R is zero. Hereafter, we will use the following notations:

$$\tilde{L}_N = \frac{L_N \pi T_c}{v_F}, \quad \tilde{N}(\varepsilon,z) = \frac{N(\varepsilon,z)}{N(0)}, \quad \tilde{\varepsilon}_g = \frac{\varepsilon_g L_N}{v_F}. (10)$$

Fig.1 shows the local density of states at the surface of the N-layer in the case $\tilde{L}_N = 1$ for several mean free paths. The impurity scattering has a great influence on the local density of states. The peak of $\tilde{N}(\varepsilon, z)$ below the superconducting gap energy becomes sharp with increasing the mean free path l in the N-side. It can be seen that the density of states has a mini-gap at intermediate concentrations of impurities and that the magnitude of the mini-gap $\tilde{\varepsilon}_g$ is sensitive to the impurity concentration.

Fig.2 shows the l dependence of the magnitude of the mini-gap $\tilde{\varepsilon}_g$ in the cases $\tilde{L}_N = 0.1, 1, 10$. We found that the DOS has a mini-gap even when $l \gg L_N$ and the magnitude of the mini-gap $\tilde{\varepsilon}_g$ is proportional to L_N/l at long mean free paths. In the case $l \ll L_N$, there also exists a mini-gap and $\tilde{\varepsilon}_g \simeq 0.26l/L_N$ in the dirty limit. The numerical result in the dirty limit is consistent with the result by Belzig et al.[2] It is also found that the mini-gap has a maximum at $l/L_N \simeq 1.386$.

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