Role of antiferromagnetic fluctuations on charge ordering and superconductivity as viewed through quantal phases

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Abstract

Using nonperturbative methods which invoke the notion of quantal (Berry) phases, we investigate the role played by antiferromagnteic fluctuations on charge ordering and superconductivity for quasi-1d and 2d spin-gapped systems.

Key words: Berry phase; superconductivity; stripe; duality

1. Introduction

Quantal phase, or Berry phase is the imaginary part of the Euclidean action which occasionaly turns up in various quantum systems. The partition function would generally bear the form,

$$Z[\phi(\mathbf{r},\tau)] = \int D[\phi]\mathrm{e}^{-S[\phi]}\mathrm{e}^{\mathrm{i}S_{\mathrm{Berry}}[\phi]}. \qquad (1)$$

The extra phase factor coming from the Berry phase will induce quantum mechanical phase interference between configurations. Particularly interesting is the case when the interference is destructive; its main effect then is to recover classical long ranged behavior which might otherwise be washed out by quantum fluctuations. A well known example appears in the Haldane gap physics of 1+1d antiferromagnets[1], where instanton Berry phases lead to the closing of the energy gap for half-integer spin systems.

Here we will be concerned with the problem of hole-doping into spin-gapped systems, with an emphasis on the role of quantal phases. The importance of quantal phases in such situations had been foreseen in works of Shankar[2] and Lee[3]. Recent developments on novel ordering such as stripes in the cuprate oxide compounds have motivated us to reexamine this issue, and we find some important differences. Below we consider the quasi-1d case, which applies to the spin-Peierls compound $CuGeO_3$ and the 2d case, with a direct relevance to the cuprate high-T_c compounds in turn.

2. Quasi-1d case

The quasi-1d spin Peierls compound is a well-defined spin-gapped system, and has the simplest nontrivial spatial structure which breaks translational symmetry. In addition its one-dimensionality enables us to apply the powerful and highly nonperturbative technique of bosonization. In these regards it should serve as an ideal starting point for studying hole-doping effects.

The Berry phase of a spin is proportional to the solid angle subtended by its imaginary-time evolution. To take full account of this phase, it is essential to incorporate a method respecting the SU(2) spin rotational symmetry. The conventional abelian bosonization, or "phase Hamiltonian" method with a fixed spin quantization axis falls short of this requirement, and we must consider its generalization to a scheme where (at half-filling) the spin quantization axis coincides with the local Neel director $\mathbf{n}(\mathbf{x},\tau)$. The task of constructing this "rotating frame bosonization " turns out to be realizable[5] with the help of Witten's nonabelian bosonization. It is best summarized in the form of a bosoniza-

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tion dictionary where we list below the bosonized expression for the uniform and staggered magnetization and the free fermion Lagrangian density $L_{\rm free}$ for the spin sector.

	abelian	rotating frame
$\mathbf{S}_{\rm k=0}$	$\mathrm{S}^z_{\rm k=0} = \frac{1}{2\pi}\partial_{\rm x}\phi_+$	$\mathbf{S}_{\rm k=0} = \frac{1}{2\pi}\partial_{\rm x}\phi_+\mathbf{n}$ $+(\text{terms related to } \partial_{\rm x}\mathbf{n})$
$\mathbf{S}_{\rm 2k_F}$	$\mathrm{S}^{\rm z}_{\rm 2k_F} \propto \cos(2{\rm k_F}x)\sin\phi_+$	$\mathbf{S}_{\rm 2k_F} \propto \cos(2{\rm k_F}x)\sin\phi_+\mathbf{n}$
action $L_{\rm free}$	$\frac{1}{4\pi^2}(\partial_\mu\phi_+)^2$	$\frac{1}{4\pi^2}(\partial_\mu\phi_+)^2 + \frac{\sin^2\phi_+}{8\pi^2}(\partial_\mu\mathbf{n})^2$ $+ \mathrm{i}(2\phi_+ - \sin 2\phi_+)q_{\rm x\tau}$

From the expression for $L_{\rm free}$, we see that for the spin-singlet case $<\phi_+>=0$ the nonlinear sigma (NLσ) model contribution to the action, describing the kinetic energy associated with the directional fluctuation of $\mathbf{n}$, together with the topological (Berry phase) term vanishes. This is to be contrasted with the usual large-S or large-N approach to quantum antiferromagnets, where both contributions are present even for singlet systems, apparently suggesting that the effect of antiferromagnetic fluctuation is overestimated by those methods.

The value of the spin phase field ϕ_+ is fixed by interactions. Applied to the spin-Peierls system modeled by a Peierls-Hubbard model, the correct singlet state is obtained for the bulk. Holes, either static or mobile, are then introduced into the system as the depletion of a unit charge. Simple energetics show that the contributions of the NLσ model part and the spin Berry phase is locally recovered in the vicinity of a hole. Under condition that the spin phase field adiabatically follows the charge depletion (or equivalently, when the charge gap is larger than the spin gap), one finds for the static case that this local recovery yields, as a low energy effective theory for the nucleated spins, the random exchange quantum Heisenberg model, fitting well with the observation of antiferromagnetism in $CuGeO_3$ with a dilute density of nonmagnetic impurities. For the mobile case, the same condition leads to a description of hole motions in terms of two massless Dirac fermions (one for each sublattice) coupled through a spin gauge field,

$$L = \bar{\psi}_A\gamma_\mu(\partial_\mu + \mathrm{i}a_\mu)\psi_A + \bar{\psi}_B\gamma_\mu(\partial_\mu - \mathrm{i}a_\mu)\psi_B, \qquad (2)$$

which coincides with Shankar's action[2]. We differ however in that a bulk spin gap is required for this action to arise. A pairing tendency arises since the two fermions have different gauge charges, as can be verified directly. Hence we see that hole induced antiferromagnetic fluctuation and the associated quantal phases are common origins for the two phenomenna discussed above. We expect this picture to be valid for a wider class of spin-gapped systems, and that nonmagnetic impurity effects can be used as probes for detecting potential superconductors.

3. 2d case

Recent experiments (e.g. [4]) seem to point towards a fairly generic tendency for spatially nontrivial order, e.g. stripes to arise in the cuprate oxide superconductors. A key question that has been raised in the physics of stripes [6] is how a nodal fermion can evolve from a stripe state which apparently enhances charge transport in the antinodal direction. Here again the quantal phase notion can be applied. Haldane[1] demonstrated how the crystal momenta carried by hedgehog defects in 2d antiferromagnets can be evaluated by regarding one spatial direction as the imaginary time axis of a 1d system. This procedure can be extended to the case where stripe order is present. We find that meron-like defects with half integer topological charge

$$Q_{xy} = \frac{1}{4\pi}\int dxdy\mathbf{n}\cdot\partial_x\mathbf{n}\times\partial_y\mathbf{n} \qquad (3)$$

which arise at the dislocation of stripes can supply the momentum $(\pi/2, \pi/2)$ to the holes[7]. Hence condensation of stripe dislocation, with the aid of antiferromagnetic fluctuation can evolve the system into one that supports nodal fermions and d-wave superconductivity. The above can be casted into a stripe-superconductor duality scheme similar to that proposed by Lee[8] but including the explicit role of antiferromagnetic fluctuations. Finally we mention that various types of spatial order can be studied along this direction by considering a variant of the SU(2) principal chiral model of the form $S = \int d\tau d^2x \mathrm{tr}[(g^{-1}\partial_\mu g)^2]$, where $g = e^{i\phi\mathbf{n}\cdot\sigma}$, in which the spatial structure is encoded into the field ϕ[7].

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