Path Integral Calculation of Superfluid ^{4}He Vortex Core Structure

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Abstract

A structure of a superfluid ^{4}He vortex core is studied with a path integral Monte Carlo method. The distribution of superfluid and normal fluid densities around the vortex core is obtained. The total particle density decreases in the vortex core region. The radius of the core is nearly equal to particle diameter. The vortex core is filled with normal fluid in the temperature region $T \simeq T_\lambda$. In low temperature region, the normal fluid component of a vortex core decreases remarkably. Consequently the number of particle density of a vortex core region becomes extremely small.

Key words: superfluid; helium-4; vortex; core;

1. Introduction

A vortex of superfluid ^{4}He is one of a typical example of a quantum vortex. In the center of the superfluid ^{4}He vortex, the superfluidity is destroyed due to divergence of superflow velocity field. We customary calls it vortex core. The order parameter of the superfluid ^{4}He is represented in a single complex vector, which is very simple compared with that of a superfluid ^{3}He.[1] But the theoretical analysis of the vortex core structure is much difficult. The reason is that a coherence length of the superfluid system is the same order of the particle radius and also the interaction potential of the particle has a hard core. There is a theoretical study of the quantum vortex structure using Gross Pitaevskii equation in the case of being assumed a weak particle-particle interaction.[2,3]

A path integral method is a one of the most powerful methods as far as dealing with strong correlated Bose condensed system.[4,5] In this paper we study a single quantized vortex and determine its core structure with using real ^{4}He particle-particle potential in a path integral Monte Carlo method. Then we calculate distributions of superfluid and normal fluid densities around the vortex.

2. Model

We confine N-particles of ^{4}He in a cylindrical container and rotate it in angular velocity ω around cylinder axis (z-axis). We impose periodic boundary condition in the z-direction. The total Hamiltonian (H) of the rotating superfluid system is given by two separated terms,

$$H = H_0 + H_{\rm s}. \tag{1}$$

The each term is given:

$$H_0 = \sum_{i=0}^{N} \frac{\boldsymbol{p}_i{}^2}{2m} + \sum_{<ij>} V(r_{ij}), \tag{2}$$

$$H_{\rm s} = \sum_{i\in\rho_{\rm s}} \frac{\hbar^2}{2m r_i{}^2} - \omega\hbar n_{\rm s}. \tag{3}$$

where r_i, $\rho_{\rm s}$ and n_i denotes radial coordinate of $i-$th particle, a superfluid density and a superfluid particle number, respectively. The term H_0 stands for the conventional kinetic energy and particle-particle potential energy of the system. On the other hand, $H_{\rm s}$ comes from superfluid component which has a quantized circulation in the rotating system. The superfluid component gives additional energy; the first term of $H_{\rm s}$ represents kinetic energy of the superfluid velocity and

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the second term means rotation effect of the system: $-\omega L_z$. Then we use the imaginary time path integral Monte Carlo method. Carrying out the numerical calculation, we employ an effective potential instead of $V(r_{ij})$, which is obtained from Aziz potential by matrix squaring method. We distinguish between a normal fluid component and a superfluid component by checking the winding number of the particle path.[6] In the Monte Carlo method, we have to generate various world line joining Canonical Emsemble. Then we try three kinds of path changes. First one is an ordinary path change for a single closed loop path. A local and a global path changes are included. Next one involves the exchange of the path which characterize Bose statistics. By this exchange, many particles join same single loop and the coherence length of the system becomes longer. Consequently Bose condensation occurs. The last one gives an effect of the switching between normal fluid $\rho_{\rm n}$ and superfluid $\rho_{\rm s}$. This path change varies winding number of the closed path. It should be noted that the evaluation of $H_{\rm s}$ is essential. The acceptance of each path change is determined by Metropolis algorithm. In the calculation used parameter is following: $N = 884, R = 10r_{\rm m}$ and z-direction length $L = 4r_{\rm m}$. We use the scaling factor $r_{\rm m} = 0.2967$nm that is a characteristic length of Aziz potential. The diameter of hard core potential region is 0.255nm. The number density of the system is that of SVP. The angular velocity ω is chosen to stabilize the single vortex state. The initial state of the particle path is a strait self-linked one.

3. Results and Discussion

Starting the path integral calculation, the winding linked path appears around a center of the cylinder axis. This should be a nucleation of the superfluid which has singly quantized circulation. Then the winding linked path grows and fills the container. In the cases of the temperature $T = 2.1$K and $T = 1.8$K, the densities of superfluid, normal fluid and also total density are shown in Fig.1 and Fig2., respectively.

The effect of wall boundary is markedly observed in normal fluid densities. A size effect is also found in each density curve. The superfluid density rapidly decreases and goes to zero at the center of the vortex. The vortex core radius is almost same as the hard core potential diameter. But the normal fluid density survives in the vortex core region. The total fluid density is also decreasing in this region. In the low temperature region, the decreasing of total density should be markedly because of decreasing the normal fluid density. The superfluid density is higher than that of bulk system. It might be caused by the term $-\omega\hbar n_{\rm s}$ in (3). The proper value of ω should be chosen. That is a future problem.

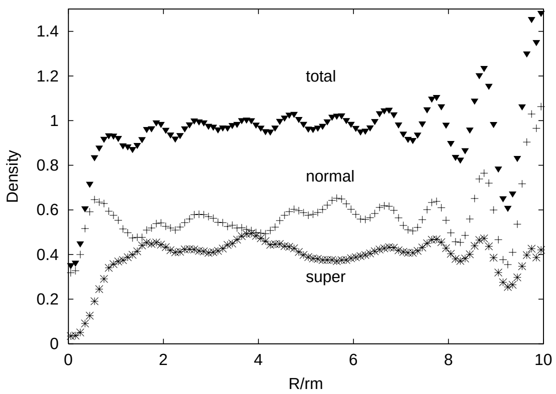

Fig. 1. Radial distribution of $\rho_{\rm s}, \rho_{\rm n}$ and ρ. In the case of $T = 2.1$K. The Density is normalized by that of VSP.

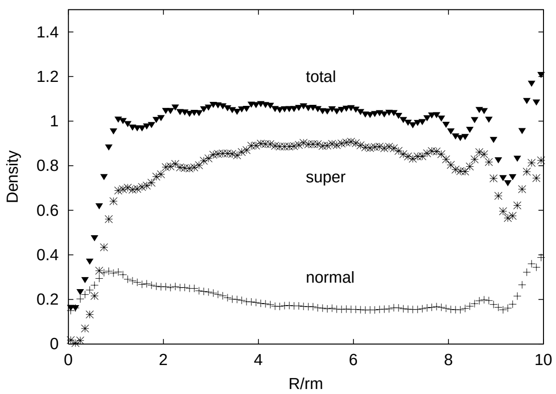

Fig. 2. Radial distribution of $\rho_{\rm s}, \rho_{\rm n}$ and ρ. In the case of $T = 1.8$K. The Density is normalized by that of VSP.

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References

[1] L. Onsager, Nuovo Cimento Suppl. **6** (1949)249.

[2] E. P. Gross, J. Math. Phys. **4** (1963)195.

[3] L. P. Pitaevskii, Soviet Phys.-JETP **13** (1961)451.

[4] D.M.Ceperley and E. L. Pollock, Phys.Rev. Lett. **56** (1986)351.

[5] M. Takahashi, J. Phys. Soc. Jpn. **55** (1986)1952.

[6] D.M.Ceperley, Rev. Mod. Phys. **67** (1995)279.