Viscoelastic theory of liquid ^{3}He in aerogel

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Abstract

Impurity scattering effect on the sound propagation in normal liquid ^{3}He in aerogel is studied from the aspect of the viscoelastic model. In this study the ^{3}He-aerogel system is modeled as a composite of a viscoelastic medium and an elastic body, the motions of which are coupled through friction between them. The dispersion relations of longitudinal and transverse sounds are obtained by solving the coupled equations of motion.

Key words: helium3; aerogel; sound

One of the most interesting properties of liquid ^{3}He is the transition from a hydrodynamic sound mode (first sound) to a collisionless sound mode (zero sound). This transition was first predicted by Landau on the basis of his theory of Fermi liquid and was observed clearly in liquid ^{3}He. It was recently discovered that liquid ^{3}He in highly porous silica aerogel does not show the first-to-zero sound transition [1,2]. The aim of this paper is to present a theory of sound propagation in such an impure Fermi liquid system as ^{3}He in aerogel from the viewpoint of the viscoelastic theory.

In the viscoelastic medium the dispersion relations for longitudinal and transverse waves $\propto \exp(i\mathbf{q}\cdot\mathbf{r}-i\omega t)$ are given as $\rho\omega^2 = K^l(\omega)q^2$ and $\rho\omega^2 = K^t(\omega)q^2$, respectively, where ρ is the mass density of the medium and $K^{l(t)}(\omega)$ is a frequency-dependent longitudinal (transverse) elastic modulus. The transverse elastic modulus is given in the complex form [3]

$$K^t(\omega) = K_\infty/(1 + i/\omega\tau). \tag{1}$$

The ω dependence of $K^t(\omega)$ is characterized by a relaxation time τ. For $\omega\tau \gg 1$, the limiting value of $K^t(\omega)$ is a real constant K_∞. Hence, the medium in this limit is elastic. For $\omega\tau \ll 1$, $K^t(\omega)$ becomes pure imaginary, i.e., $K^t(\omega) \simeq -iK_\infty\omega\tau$; therefore the medium behaves like a viscous liquid with the shear viscosity $\eta = K_\infty\tau$. The longitudinal elastic modulus can be written as [3,4]

$$K^l(\omega) = K_0 + \frac{4}{3}K^t(\omega), \tag{2}$$

where K_0 is a real constant. Equation (2) shows that the longitudinal sound propagates with a velocity $c_1 = (K_0/\rho)^{1/2}$ in the limit $\omega\tau \to 0$ and with a velocity $c_0 = [(K_0+\frac{4}{3}K_\infty)/\rho]^{1/2}$ in the limit $\omega\tau \to \infty$, corresponding to the first sound and the zero sound, respectively.

The relaxation time τ in pure ^{3}He is conventionally written as τ_η, which is the viscous relaxation time due to mutual collisions between ^{3}He quasiparticles. In the ^{3}He-aerogel system, impurity scattering contributes to τ as well; then τ should be modified as $\tau^{-1} = \tau_\eta^{-1} + \tau_a^{-1}$ [1], where the contribution from aerogel-quasiparticle collisions is denoted by τ_a^{-1}.

Nomura *et al.* [1] have recently analyzed their experimental data of longitudinal sound attenuation using the viscoelastic model with the modified τ. They found that the temperature dependence of the observed attenuation cannot be reproduced by such a model. This does not mean the breakdown of the viscoelastic theory. In fact, as shall be shown below, the experimental result can be explained well by the viscoelastic model if we take into account the effect of friction between liquid and aerogel in addition to the modification of τ.

The effect of the friction can easily be incorporated into the theory by considering the collision integral in

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the Landau transport equation. When aerogel is at rest (the effect of aerogel motion shall be taken into account later), the linearized collision integral for impurity scattering can be written as [5]

$$I_{\rm imp} = -2\pi \sum_{\mathbf{k}'} W_{\mathbf{k},\mathbf{k}'} \delta(\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'})(\delta n'_{\mathbf{k}} - \delta n'_{\mathbf{k}'}), \qquad (3)$$

where $\epsilon_{\mathbf{k}}$ is the equilibrium energy of a quasiparticle with momentum $\mathbf{k}$, $\delta n'_{\mathbf{k}}$ is the linear deviation of the distribution function from its local equilibrium value and $2\pi W_{\mathbf{k},\mathbf{k}'}\delta(\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'})$ represents the probability of elastic scattering from the state $\mathbf{k}$ to the state $\mathbf{k}'$. Since the impurity scattering does not conserve the quasiparticle momentum, it gives rise to momentum transfer between liquid and aerogel. Using eq. (3), the net momentum transfer (per unit time and per unit volume) from liquid to aerogel can be calculated as [5]

$$-2\int \frac{{\rm d}^3\mathbf{k}}{(2\pi)^3}\mathbf{k} I_{\rm imp} = \frac{1}{\tau_{\rm tr}}(1 + F_1^s/3)\rho\dot{\mathbf{u}}, \qquad (4)$$

where $\tau_{\rm tr}$ is the transport mean free time, F_1^s the p-wave Landau parameter and $\mathbf{u}$ the displacement vector of liquid ($\dot{\mathbf{u}}$ is the velocity field of liquid).

Equation (4) is proportional to the velocity $\dot{\mathbf{u}}$ and can be regarded as friction exerted on aerogel. When aerogel is in motion, the friction should be proportional to the relative velocity $\dot{\mathbf{u}} - \dot{\mathbf{u}}_a$, where $\mathbf{u}_a$ is the displacement vector of aerogel.

The friction causes coupled motion of liquid and aerogel. Assuming aerogel to be an elastic body characterized by longitudinal and transverse elastic moduli, K_a^l and K_a^t, we obtain the following coupled equations of motion of $\mathbf{u}$ and $\mathbf{u}_a$:

$$\rho\omega^2\mathbf{u}^{l,t} = K^{l,t}(\omega)q^2\mathbf{u}^{l,t} - \frac{i\omega}{\tau^*_{\rm tr}}\rho(\mathbf{u}^{l,t} - \mathbf{u}_a^{l,t}), \qquad (5)$$

$$\rho_a\omega^2\mathbf{u}_a^{l,t} = K_a^{l,t}q^2\mathbf{u}_a^{l,t} + \frac{i\omega}{\tau^*_{\rm tr}}\rho(\mathbf{u}^{l,t} - \mathbf{u}_a^{l,t}), \qquad (6)$$

where $\mathbf{u}^{l,t}$ and $\mathbf{u}_a^{l,t}$ denote longitudinal and transverse parts of the displacement vectors, ρ_a is the mass density of aerogel and $\tau^*_{\rm tr} = \tau_{\rm tr}/(1 + F_1^s/3)$. It follows from eqs. (5) and (6) that the dispersion relations for longitudinal and transverse sounds can be written as

$$\rho\omega^2 + \frac{i\rho\omega}{\tau^*_{\rm tr}}\frac{\rho_a\omega^2 - K_a^{l,t}q^2}{\rho_a\omega^2 - K_a^{l,t}q^2 + i\rho\omega/\tau^*_{\rm tr}} = K^{l,t}(\omega)q^2. \qquad (7)$$

Let us compare this result with the experiment [1,2] on longitudinal sound. The sound frequency ($\omega/2\pi \sim$ 15 MHz) used in the experiment [1,2] is low enough so that the system is in the impurity-dominated hydrodynamic regime. This allows us to expand eq. (7) in powers of $\omega\tau$ and $\omega\tau_{\rm tr}$. Moreover, since the observed sound velocity is much larger than that of bare aerogel, $(K_a^l/\rho_a)^{1/2}$ [2], we may neglect $K_a^l q^2$ in eq. (7). Thus,

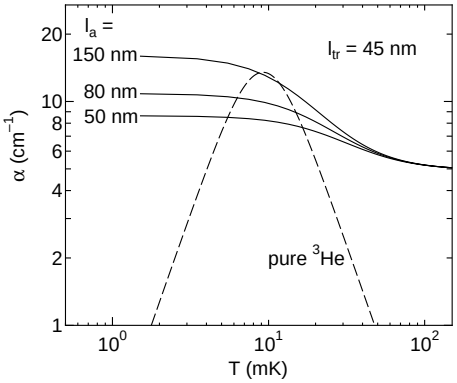

Fig. 1. Temperature dependence of the attenuation of 15 MHz longitudinal sound in the ^{3}He-aerogel system (solid lines) and in pure ^{3}He (dashed line) at a pressure of 16 bar. The parameter values for ^{3}He are taken from Refs. [4] and [6]. The density of aerogel is $\rho_a = 0.044$ g/cm^3 [2].

keeping only the leading order terms in $\omega\tau$ and $\omega\tau_{\rm tr}$, we obtain the following approximate dispersion relation applicable to the actual ^{3}He-aerogel system used in the experiment:

$$\frac{\omega^2}{q^2} = \frac{\rho c_1^2}{\rho + \rho_a}\left[1 - \frac{c_0^2 - c_1^2}{c_1^2}i\omega\tau - \frac{\rho_a^2}{\rho(\rho + \rho_a)}i\omega\tau^*_{\rm tr}\right]. \qquad (8)$$

We see from eq. (8) that the longitudinal sound velocity is given by $[\rho c_1^2/(\rho + \rho_a)]^{1/2}$. It follows that aerogel moves together with liquid and gives extra inertia in liquid oscillation. The observed temperature dependence of attenuation α [1,2] is well reproduced by that calculated from eq. (8) with $l_a = v_F\tau_a = 80$ nm and $l_{\rm tr} = v_F\tau_{\rm tr} = 45$ nm (see Fig. 1).

In conclusion, the propagation of longitudinal sound in the ^{3}He-aerogel system has been shown to be well described by the viscoelastic theory taking into account simultaneous motion of aerogel. The study of transverse sound shall be reported elsewhere.

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