A massive gauge mechanism of the spin glasses

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Abstract

We have introduced the microscopic theory of the spin glass, which is gauge-invariant, taking into account the mixing the spin and the chirality through mediating by the gauge fields.

Key words: spin glass; spin chirality; the gauge fields

1. Introduction

Experiments have provided convincing evidence that spin-glass (SG) magnets exhibit a phase transition at a finite temperature in the three-dimensional Heisenberg SG [1]. In apparent contrast to experiments, numerical simulations have indicated that the standard spin-glass order occurs only at zero temperature in the three-dimensional Heisenberg SG [2,3].

In order to solve this apparent puzzle, Kawamura [4] has proposed a chirality driven mechanism through the Monte Carlo simulations, and have indicated the importance of the mixing between the spin and the chirality [5]. In the present study, we will propose the microscopic theory of the spin glass, which is gauge-invariant, taking into account the mixing between the spin and the chirality through mediating by the gauge fields.

2. A model system

We will consider the effect of the gauge field fluctuation, A_μ^a, in three-dimensional Heisenberg model. Based on the important idea [6], it has been proposed that the hedgehog-like fluctuation in three-dimensional system is specified by rigid-body rotation, which is related to gauge fields of SO(4) symmetry for S^3 [7-9]. Thus it is thought that non-linear gauge fields A_μ^a introduced by the hedgehog-like fluctuations have a local SO(4) symmetry. The SO(4) quadruplet fields A_μ^a are spontaneously broken through the Higgs mechanism similar to the way in which three-dimensional Heisenberg system is broken around the hedgehog-like fluctuation (cluster). In order to discuss the gauge field fluctuation, we introduce the Lagrangian density as follows,

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\left(\partial_i S^j - g_1 \varepsilon_{ijk} A_i^a S^k\right)^2 \\ &-\frac{1}{4}\left(\partial_\nu A_\mu^a - \partial_\mu A_\nu^a + g_3 \varepsilon_{abc} A_\mu^b A_\nu^c\right)^2 \\ &+\frac{1}{2}\left(\partial_\mu \phi_a - g_4 \varepsilon_{abc} A_\mu^b \phi_c\right)^2 \\ &-\lambda^2\left(\phi_a\phi_a - \mu^2\right)^2 . \end{aligned} \tag{1}$$

After the symmetry breaking $\langle 0|\phi_a|0\rangle = \langle 0,0,0,\mu\rangle$ or $(\langle 0|\phi_a|0\rangle = \langle 0,0,0,-\mu\rangle)$, we can obtain the effective Lagrangian density,

$$\begin{aligned}\mathcal{L}_{\text{eff}} = &\frac{1}{2}\left(\partial_i S^j - g_1 \varepsilon_{ijk} A_i^a S^k\right)^2 \\ &-\frac{1}{4}\left(\partial_\nu A_\mu^a - \partial_\mu A_\nu^a + g_3 \varepsilon_{abc} A_\mu^b A_\nu^c\right)^2 \\ &+\frac{1}{2}\left(\partial_\mu \phi_a - g_4 \varepsilon_{abc} A_\mu^b \phi_c\right)^2 \\ &+\frac{1}{2}{m_1}^2\left[\left(A_\mu^1\right)^2 + \left(A_\mu^2\right)^2 + \left(A_\mu^3\right)^2\right] \\ &+m_1\left[A_\mu^1 \partial_\mu \phi_2 - A_\mu^2 \partial_\mu \phi_1\right]\end{aligned}$$

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$$
\begin{aligned}
&+m_1\left[A_\mu^2\partial_\mu\phi_3 - A_\mu^3\partial_\mu\phi_2\right] \\
&+m_1\left[A_\mu^3\partial_\mu\phi_1 - A_\mu^1\partial_\mu\phi_3\right] \\
&+g_4 m_1\Big\{\phi_4\left(A_\mu^1\right)^2 + \left(A_\mu^2\right)^2 + \left(A_\mu^3\right)^2 \\
&-A_\mu^4\left[\phi_1 A_\mu^1 + \phi_2 A_\mu^2 + \phi_3 A_\mu^3\right]\Big\} - \frac{{m_2}^2}{2}\left(\phi_4\right)^2 \\
&-\frac{{m_2}^2 g_4}{2m_1}\phi_4\left(\phi_a\right)^2 - \frac{{m_2}^2{g_4}^2}{8{m_1}^2}\left(\phi_a\phi_a\right)^2, \qquad (2)
\end{aligned}
$$

where S^j is the spin, $m_1 = \mu g_4$, and $m_2 = 2\sqrt{2}\lambda\mu$. The effective Lagrangian describes three massive gauge fields A_μ^1, A_μ^2, and A_μ^3, and one massless gauge field A_μ^4. Because masses of A_μ^1, A_μ^2, and A_μ^3 are formed through the Higgs mechanism by introducing the hedgehog-like fluctuation, the fields A_μ^1, A_μ^2, andA_μ^3 exist around the hedgehog-like fluctuation within the length of $\sim \frac{1}{m_1} \equiv R_C$. From the first term in eq. (2), the spin S^i is much distorted within the length of $\sim R_C$ around the hedgehog-like fluctuation. Furthermore, the spin will be distorted in the long-range interaction by the massless gauge field A_μ^4. When $S(i)$, $S(j)$ and $S(k)$ are spins on triangle sites i, j, and k within $\sim \frac{4\pi}{3}{R_C}^3\left(\tilde{i}\right)$ around the hedgehog-like fluctuation at the site $\tilde{i}$, the chiral spin parameter $q_{\tilde{i}}$ is introduced as

$$
q_{\tilde{i}} \equiv \sum_{(i,j,k)\in\frac{4\pi}{3}{R_C}^3\left(\tilde{i}\right)} S(i)\cdot\left(S(j)\times S(k)\right),
$$

where (i,j,k) are local triplet sites of spins. When the hedgehog-like fluctuation is located at the position $r_{\tilde{i}}$ and $|r - r_{\tilde{i}}| \gg \frac{1}{m_1} \sim R_C$ is assumed, the massless gauge field $A_\mu^4(r, r_{\tilde{i}})$ at the position r is represented as $A_\mu^4(r, r_{\tilde{i}}) \propto \frac{q_{\tilde{i}}}{|r - r_{\tilde{i}}|}$. Thus, we can introduce the interaction between the chiral spin-disordered hedgehog-like solitons at positions $r_{\tilde{i}}$ and $r_{\tilde{j}}$ as $V_{\tilde{i}\tilde{j}} \propto \frac{\left(q_{\tilde{i}}\cdot q_{\tilde{j}}\right)}{|r_{\tilde{i}} - r_{\tilde{j}}|}$. For the mean-field approximate, it is assumed that $V_{\tilde{i}\tilde{j}}$ describes N hedgehog-like soliton's interaction, which mediated by the massless A_μ^4 field, in pair $\left(\tilde{i},\tilde{j}\right)$ via infinite-range Gaussian-random interaction for simplifying discussions,

$$
P\left(V_{\tilde{i}\tilde{j}}\right) = \frac{1}{\left(2\pi\left\langle {V_{\tilde{i}\tilde{j}}}^2\right\rangle\right)^{\frac{1}{2}}}\exp\left(\frac{-{V_{\tilde{i}\tilde{j}}}^2}{2\left\langle {V_{\tilde{i}\tilde{j}}}^2\right\rangle}\right). \qquad (3)
$$

We can define the order parameter $\bar{G} \equiv \int_0^1 \mathrm{d}x Q(x)$, where $Q(x)$ is the Parisi order parameter and is derived from $Q^{\alpha\beta} \equiv \left\langle q_{\tilde{i}}^\alpha q_{\tilde{i}}^\beta\right\rangle$ with replica indices α and β, in Parisi's theoretical formula [10]. In the temperature region below $\frac{\left(\left\langle {V_{\tilde{i}\tilde{j}}}^2\right\rangle\right)^{\frac{1}{2}}}{k_B}$, we got the order parameter $\bar{G} \neq 0$ and $\langle q_{\tilde{i}}^\alpha\rangle = 0$, which corresponds to the chiral spin-glass phase, within the mean field approximate.

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