Peierls Instability of the Quasi-One-Dimensional Bose-Fermi Mixed Gas

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Abstract

The mixtures of quantum degenerate bose and fermi gases in highly anisotropic traps at zero temperature are studied. It is found that under some conditions the system becomes unstable with respect to the spontaneous formation of the collective mode with wave-vector $2k_F$ (k_F is a Fermi wave-vector) by exploring the Bogoliubov phonon spectrum. This type of instability is the analogue of Peierls instability in quasi-one-dimensional conductors.

Key words: Bose-Einstein condensate; degenerate fermi gas; Peierls instability

1. Introduction

The development in the trapping techniques of atoms has allowed the achievement of gases which can be considered as one-dimensional (1D). Recently, the realization of Bose-Einstein condensation (BEC) in 1D systems has been reported at MIT [1]. In this paper, we study the mixtures of spin-polarized bose and fermi atoms of mass m, trapped in an external potential $U(\mathbf{r}) = \frac{1}{2}m\{\omega_t^2(x^2+y^2)+\omega_l^2 z^2\}$ ($\omega_t \gg \omega_l$), at zero temperature.

The system can be modeled by the following Hamiltonian:

$$\begin{aligned} H = &\int \mathrm{d}^3 r \hat{\phi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) - \mu_b \right) \hat{\phi}(\mathbf{r}) \\ &+ \int \mathrm{d}^3 r \hat{\psi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) - \mu_f \right) \hat{\psi}(\mathbf{r}) \\ &+ \frac{2\pi\hbar^2 a_{bb}}{m} \int \mathrm{d}^3 r \hat{\phi}^\dagger(\mathbf{r})\hat{\phi}^\dagger(\mathbf{r})\hat{\phi}(\mathbf{r})\hat{\phi}(\mathbf{r}) \\ &+ \frac{4\pi\hbar^2 a_{bf}}{m} \int \mathrm{d}^3 r \hat{\phi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\phi}(\mathbf{r}), \end{aligned} \qquad (1)$$

where $\hat{\phi}^{(\dagger)}(\mathbf{r})$ and $\hat{\psi}^{(\dagger)}(\mathbf{r})$ are bosonic and fermionic annihilation (creation) operators. $a_{bb}(>0)$ and a_{bf} are the s-wave scattering lengths for the repulsive boson-boson and boson-fermion scatterings. The chemical potentials μ_b and μ_f are determined from the condensed boson and fermion numbers N_0 and N_F

It is known that the phonon-electron interaction in quasi-1D conductors produces a giant Kohn anomaly in the phonon spectrum and causes a Peierls instability [2]. Turning to atomic bose-fermi mixtures, the interaction between Bogoliubov phonon and fermion would give an analogous effect. In the quasi-1D regime where any longitudinal excitations have lower energy than the transverse excitation energy $\hbar\omega_t$, the transverse wave function of bosons and fermions remain the ground state in 2D harmonic oscillator potential: $\phi_{00}^{ho}(x,y)$. In this case, the interaction between particles acquires 3D character and the effective 1D coupling constant will be characterized by $g_{bb,bf}^{1D} = 2\hbar\omega_t a_{bb,bf}$[3].

The boson operator $\hat{\phi}$ is separated into the condensate wave function $\sqrt{N_0}\varphi_0(z)\phi_{00}^{ho}(x,y) = \langle\hat{\phi}\rangle$ and fluctuation part $\hat{\phi}'$. In the Bogoliubov approximation $\hat{\phi}'$ is expressed in terms of quasi-particle operators $\hat{\beta}_\lambda$ and $\hat{\beta}_\lambda^\dagger$ through $\hat{\phi}'(\mathbf{r}) = \sum_\lambda \{u_\lambda(z)\hat{\beta}_\lambda - v_\lambda^*(z)\hat{\beta}_\lambda^\dagger\}\phi_{00}^{ho}(x,y)$. On the Bogoliubov phonon spectrum we consider only the Bogoliubov phonon self-energy arisen from

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the coupling to fermion-hole pair excitations (neglecting Hartree potential), because we are interested in the Kohn anomaly. Using the eigenenergy $\epsilon_n^{ho} = \hbar\omega_z(n+\frac{1}{2})(n=0,1,2,\cdots)$ and wave function ϕ_n^{ho} in 1D harmonic oscillator potential, we express the quasiparticle amplitudes u_λ and v_λ through $u_\lambda(z) = \sum_n u_n^\lambda \phi_n^{ho}(z)$, $v_\lambda(z) = \sum_n v_n^\lambda \phi_n^{ho}(z)$. We can obtain the excitation energies ϵ_λ solving the equations for the coefficients u_n^λ and v_n^λ,

$$\sum_n \left[(\epsilon_n^{ho} - \epsilon_\lambda)\delta_{m,n}u_n^\lambda + \hbar\Pi_{mn}(\epsilon_\lambda)(u_n^\lambda - v_n^\lambda)\right] = 0 \quad (2)$$

$$\sum_n \left[(\epsilon_n^{ho} + \epsilon_\lambda)\delta_{m,n}v_n^\lambda + \hbar\Pi_{mn}(\epsilon_\lambda)(v_n^\lambda - u_n^\lambda)\right] = 0. \quad (3)$$

The Bogoliubov phonon self energy $\hbar\Pi_{mn}$ is given by

$$\hbar\Pi_{mn}(\epsilon_\lambda) = (g_{bf}^{1D})^2 N_0 \sum_{ph}\left[\frac{\langle mh|p0\rangle\langle 0p|hn\rangle}{\epsilon_\lambda - \epsilon_p^{ho} + \epsilon_{h,f}^{1D}} - \frac{\langle mp|h0\rangle\langle 0h|pn\rangle}{\epsilon_\lambda - \epsilon_h^{ho} + \epsilon_{p,f}^{1D}}\right]. \quad (4)$$

The matrix elements can be calculated as $\langle mh|p0\rangle = \int dz \phi_m^{ho}\phi_h^{ho}\phi_p^{ho}\varphi_0$, where the fermionic wave function has been replaced by ϕ_n^{ho} and $p(>n_F)/h(<n_F)$ denote particle/hole states. The n_F is the number of fermions in the last occupied state.

2. Peierls instability condition

We restrict the parameter region: $\omega_t > \mu_f^{1D} > \mu_b^{1D} > \hbar\omega_l$ where $\mu_{b,f}^{1D} = \mu_{b,f} - \hbar\omega_t$. Under these conditions we give several approximations. i) Condensate wave function in Thomas-Fermi regime ($\mu_b^{1D} > \hbar\omega_l$) : $\varphi_0 \propto \left[1 - \frac{z^2}{z_{TF}^2}\right]^{1/2}\theta(z_{TF} - |z|)$, $z_{TF} = \sqrt{2}a_{ho}(3N_0\beta/4\sqrt{2})$ ($\beta = mg_{bb}^{1D}a_{ho}/\hbar^2$ where $a_{ho} = \sqrt{\hbar/m\omega_z}$ is a longitudinal oscillator length). ii) Asymptotic expansion of ϕ_n^{ho} in the middle of the trap for the higher nodal modes $n \sim n_F \gg 1 : \phi_n^{ho}(z) \propto \cos\left(k_n z - \frac{n\pi}{2}\right)$, $|z| < \sqrt{2n}a_{ho}$ ($k_n = \sqrt{2n}/a_{ho}$). iii) In case we discuss the higher nodal modes ($\sim n_F$) the negative energy amplitude v_n^λ of Eq.(2) can be neglected.

On the basis of these approximations we can calculate the matrix elements as

$$\langle mh|p0\rangle \propto C_p^{mh}\frac{J_1(z_{TF}K_p^{mh})}{z_{TF}K_p^{mh}} + C_h^{mp}\frac{J_1(z_{TF}K_h^{mp})}{z_{TF}K_h^{mp}} + C_{ph}^{m}\frac{J_1(z_{TF}K_{ph}^{m})}{z_{TF}K_{ph}^{m}} + C^{mph}\frac{J_1(z_{TF}K^{mhp})}{z_{TF}K^{mhp}}, \quad (5)$$

where Cs are constants and Ks are wave-vector differences between initial and finial states, e.g., $K_p^{mh} = k_m + k_h - k_p$. In the case of $|x| \gg 1$ the asymptotic expansion of the Bessel function is obtained as $J_1(x)/x \sim |x|^{-3/2}$, so that the only term with $|x| \leq \mathcal{O}(1)$ contributes to the matrix element. Under the restriction of wave vectors $k_h \leq k_F$, $k_p > k_F$ with $k_n \sim 2k_F$ ($k_F = \sqrt{2n_F}/a_h o$), the main contributions are the second and third terms of Eq. (5). In order to simplify the algebra even further we take: $\frac{J_1(x_1)}{x_1}\frac{J_1(x_2)}{x_2} \simeq \frac{1}{4}\theta(1-|x_1|)\theta(1-|x_1-x_2|)$, and replace the sum $\sum_{p,h}$ by $\int dp \int dh$ in the continuum limit.

Finally, we can get the eigenenergy for a collective state with $k_n = 2k_F$ ($v_F = \hbar k_F/m$, $\epsilon_F = \hbar^2 k_F^2/2m$),

$$\hbar\omega_\lambda = 4\epsilon_F - \frac{(g_{bf}^{1D})^2 n_0(0)}{2\pi\hbar v_F}\ln|4k_F z_{TF}|. \quad (6)$$

This equation is no longer correct for the system which satisfies the condition

$$1 < \frac{\zeta}{4}\frac{v_B^2}{v_F^2}\ln|4k_F z_{TF}| \quad (7)$$

where $v_B = \sqrt{g_{bb}^{1D}n_0(0)/m}$ and $\zeta = (g_{bf}^{1D})^2/\pi g_{bb}^{1D}\hbar v_F$. Eq.(7) is the condition for the Peierls instability where BEC may occur at a $k_n \simeq 2k_F$ state in addition to the ordinary $n=0$ state.

Lets us look at the values of the parameters for actual experimental conditions. We can express the velocity ratio and dimensionless coupling constant as $\frac{v_B^2}{v_F^2} = \frac{1}{4N_f}\left(3N_0\frac{\omega_t}{\omega_l}\frac{a_{bb}}{a_{ho}}\right)^{2/3}$, $\zeta = \sqrt{\frac{2}{\pi^2 N_f}\frac{\omega_t}{\omega_l}}\frac{a_{bf}^2}{a_{bb}a_{ho}}$. A possible candidate for Peierls instability does is the rubidium isotope system: ^{87}Rb$-^{84}$Rb mixtures. We take for the scattering lengths $a_{bb} = 5.3$[nm] and $a_{bf} = 29.1$[nm] given in [4] and trapping frequencies of $\omega_l = 2\pi \times 10$[Hz] and $\omega_t = 2\pi \times 15$[kHz]. We can find the parameters $N_f = 10^3$, $N_0 = 2\times 10^4$, ($v_B^2/v_F^2 = 0.67$, $\zeta = 0.99$), which satisfied with the condition of Eq. (7).

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