Finite-size effects on the thermal conductivity of ^{4}He near T$_\lambda$

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Abstract

We present results of a renormalization-group calculation of the thermal conductivity of confined ^{4}He in a $L^2 \times \infty$ geometry above and at T_λ within model F with Dirichlet boundary conditions for the order parameter. We assume a heat flow parallel to the boundaries which implies Neumann boundary conditions for the entropy density. No adjustable parameters other than those known from bulk theory are used. Our theoretical results are compared with experimental data by Kahn and Ahlers.

Key words: critical phenomena; finite-size effects; helium4; thermal conductivity

1. Introduction

The theory of finite-size effects near phase transitions is an area of active research. Well suited for a comparison between theory and experiment is the superfluid transition of ^{4}He [1]. So far, however, primarily *static* properties have been investigated. Very little is known about finite size effects on *dynamic* quantities. In particular no theoretical prediction exists for the validity of dynamic finite-size scaling along the lambda line of ^{4}He. Here we present some results of the first renormalization-group calculation of finite-size effects on the thermal conductivity of ^{4}He above and at T_λ and compare our results with experimental data by Kahn and Ahlers [2].

2. Theory

Our calculations are based on model F [3] which is defined by

$$\dot{\psi}_0 = -2\Gamma_0 \frac{\delta H}{\delta \psi_0^*} + i g_0 \psi_0 \frac{\delta H}{\delta m_0} + \Theta_\psi \ , \quad (1)$$

$$\dot{m}_0 = \lambda_0 \nabla^2 \frac{\delta H}{\delta m_0} + g_0 \nabla \mathbf{j}_s^0 + W_0 + \Theta_m \ , \quad (2)$$

$$H = \int d^3\mathbf{r} \Big(\frac{1}{2} r_0 |\psi_0|^2 + \frac{1}{2} |\nabla \psi_0|^2 + \tilde{u}_0 |\psi_0|^4 + \frac{1}{2} \chi_0^{-1} m_0^2 + \gamma_0 m_0 |\psi_0|^2 \Big) \ , \quad (3)$$

where

$$\mathbf{j}_s^0(\mathbf{r},t) \equiv \mathrm{Im}(\psi_0^*(\mathbf{r},t) \nabla \psi_0(\mathbf{r},t)) \ , \quad (4)$$

$$r_0 = r_{0c} + a_0 \tilde{t} \ , \tilde{t} = (T - T_\lambda)/T_\lambda \ . \quad (5)$$

We consider a rectangular $L^2 \times \widetilde{L}$ box geometry and assume a stationary heat current Q in the z direction which is generated by a heat source in the bottom plane $z = -\widetilde{L}/2$ and absorbed by a sink in the top plane $z = \widetilde{L}/2$,

$$W_0(\mathbf{r}) = Q[\delta(z + \widetilde{L}/2) - \delta(z - \widetilde{L}/2)] \ . \quad (6)$$

We impose Dirichlet boundary conditions for the order parameter ($\psi_0 = 0$) and Neumann boundary conditions for the entropy density m_0 (vanishing spatial derivatives perpendicular to the sidewalls). Eventually we let $\widetilde{L} \to \infty$ and define the superfluid current $\mathbf{j} = \lim_{\widetilde{L}\to\infty} < \mathbf{j}_s^0 >$ in the stationary state. We are interested in the finite-size effect on the thermal conductivity

$$\lambda_T(\tilde{t}, L) = \lambda_0 \left[1 + \frac{g_0}{L^2} \lim_{Q \to 0} \frac{\partial}{\partial Q} \int_{L^2} dx\, dy j_z \right]^{-1} \qquad (7)$$

where j_z is the z component of $\mathbf{j}$. We have calculated $\lambda_T(\tilde{t}, L)$ analytically for $\tilde{t} \geq 0$ up to one-loop order employing the minimal renormalization approach at fixed dimension $d = 3$ [4] and using the effective parameters known from bulk theory [5]. We neglect possible nonscaling contributions due to cutoff effects and van der Waals forces [6].

3. Results

In Fig. 1 we present our prediction of $\lambda_T(0, L)$ at saturated vapor pressure. It agrees reasonably well with the experimental result for holes $2\mu m$ in diameter [2].

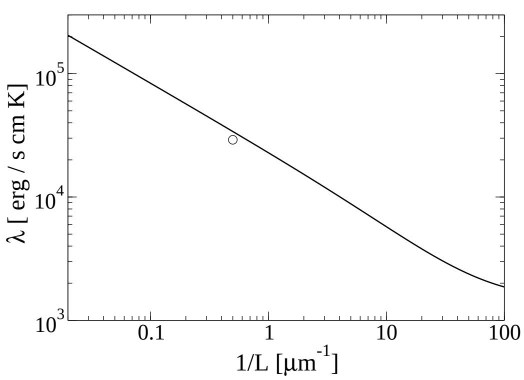

Fig. 1. Theoretical prediction for the thermal conductivity, Eq. (7), at $T = T_\lambda$ as a function of L^{-1}. The circle represents the experimental result by Kahn and Ahlers [2] for holes $2\mu m$ in diameter.

In Fig. 2 we plot our prediction for the relative deviation

$$\Delta\lambda = \frac{\lambda_b(\tilde{t}) - \lambda_T(\tilde{t}, L)}{\lambda_b(\tilde{t})} \qquad (8)$$

from the bulk thermal conducivity $\lambda_b(\tilde{t}) \equiv \lambda_T(\tilde{t}, \infty)$ at saturated vapor pressure for $L = 2\mu m$ in the regime $\xi \ll L$ (solid line) where ξ is the bulk correlation length above T_λ. It agrees reasonably well with the experimental data [2] in this regime.

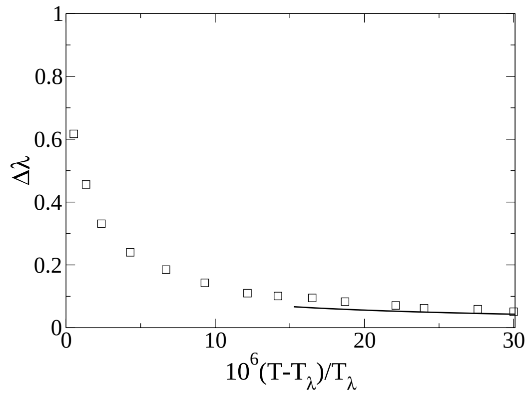

Fig. 2. Relative deviation $\Delta\lambda$, Eq. (8), versus reduced temperature. Solid line: Theoretical prediction for $L = 2\mu m$ in the regime $\xi \ll L$. Squares: Representative set of data taken from Fig. 2 of Ref. [2].

Our analytical theoretical expression for $\lambda_T(\tilde{t}, L)$ will be presented elsewhere [7]. It will enable us to make quantitative predictions for the finite-size scaling function of λ_T and for the pressure dependence of the finite-size effects along the lambda line. This will provide the basis for testing the range of validity of universal dynamic finite-size scaling.

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