Instability of AB interfaces of different shapes in rotating ^{3}He

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Abstract

In our experiments on rotating superfluid ^{3}He the interface between the A and B phases is stabilized in a magnetic field. With decreasing current in the barrier magnet, the shape of the interface changes from almost flat and perpendicular to the sample axis to a ring of A phase which coats the outer sample boundary, with B phase in the center. The influence of the shape of the interface on the critical velocity of its shear-flow instability is investigated.

Key words: superfluidity; helium3; hydrodynamics; shear-flow

The shear-flow instability of the interface between two fluid layers is well-known in classical hydrodynamics. It is attributed to many natural phenomena, such as the formation of waves by wind. The phenomenon is usually explained in terms of the Kelvin-Helmholtz (KH) instability for ideal non-viscous fluids. Recently the first example of the ideal case was discovered in rotating superfluid ^{3}He [1]. Here the phase boundary between ^{3}He-A and ^{3}He-B is stabilized with a magnetic field and lies almost perpendicular to the axis of the cylindrical sample [2]. When the sample is rotated around its axis with angular velocity Ω, the A phase fills with vortex lines. Its superfluid velocity $v_{\rm sA}$ mimics the solid-body rotation of the normal component: $v_{\rm sA} \approx v_{\rm n} = \Omega r$, where r is the radial distance from the rotation axis. In contrast, the B phase remains vortex-free, $v_{\rm sB} = 0$, and a large counterflow velocity is formed, $v_{\rm sB} - v_{\rm n} = -\Omega r$. When its magnitude reaches the critical value for the AB interface, $v_{\rm cAB}$, the instability develops, producing corrugations on the interface. The experimentally observed fingerprint of the instability is the penetration of some vorticity to the B phase. Unlike instabilities in classical systems, whose description is plagued by difficulties because of dissipation, the inviscid nature of the vortex-free counterflow state in superfluids allows a straightforward theoretical formulation of the threshold $v_{\rm cAB}$, in a manner similar to the ideal KH model [1].

The instability happens at the outer sample boundary where the counterflow velocity is at maximum, *ie.* $v_{\rm cAB} = \Omega_{\rm cAB} R$. Here $\Omega_{\rm cAB} \sim 1\,$rad/s is the measured critical angular velocity and $R = 3\,$mm is the sample radius. The spatial extent of the instability on the AB interface is determined by its wavelength $\lambda = 4\pi\sigma_{\rm AB}/\rho_{\rm sB} v_{\rm cAB}^2$, where $\sigma_{\rm AB}$ is the surface tension and $\rho_{\rm sB}$ is the B-phase superfluid density. Depending on experimental conditions, λ varies in the range 0.25 – 1.7 mm.

The spatial aspects of the instability can be checked by studying its development for interfaces of different shapes. The shape of the interface is determined by the competition of several energy contributions: (a) Difference in the magnetic energies of the two phases $\int_{\rm A} (1/2)(\chi_{\rm A} - \chi_{\rm B})(H_{\rm AB}^2 - H_{\rm b}^2)\, dV$, where $\chi_{\rm A,B}$ are the susceptibilities of the two phases, $H_{\rm AB}$ is the thermodynamic equilibrium value of the magnetic field of the AB transition at given temperature and pressure, $H_{\rm b}$ is the applied magnetic field and the integral is taken over

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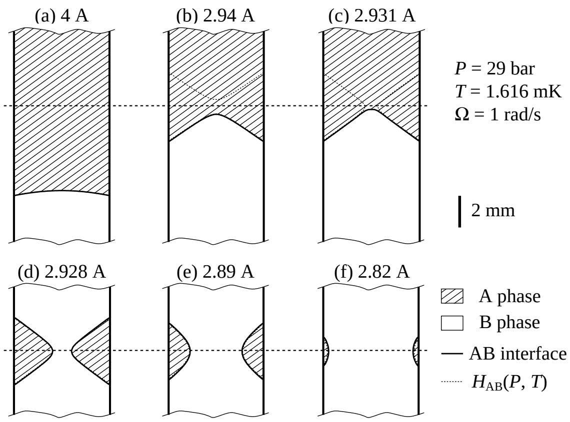

Fig. 1. Calculated shapes of the AB interface at different currents $I_{\rm b}$ in the barrier solenoid (marked above each vertical cross-section of the cylindrical sample): singly-connected *(a–c)* and toroidal *(d–f)* A-phase configurations. The horizontal dashed line marks the center plane of the barrier magnet. Parameter values of the calculations: $\chi_{\rm A} - \chi_{\rm B} = 5.25 \cdot 10^{-8}$ cgs, $H_{\rm AB} = 2760\,$Gs, $\sigma_{\rm AB} = 9.34 \cdot 10^{-6}\,$erg/cm^2, $\theta_{\rm AB} = 68°$, $\rho_{\rm sA} = \rho_{\rm sB} = 2.28 \cdot 10^{-2}\,$g/cm^3, $\Omega_{\rm cA} = 0.15\,$rad/s. The B phase is vortex-free. The calculated magnetic field profile of the barrier magnet is scaled with 1.063 to adjust the A→B transition above the magnet to the measured current 2.929 A in Fig. 2.

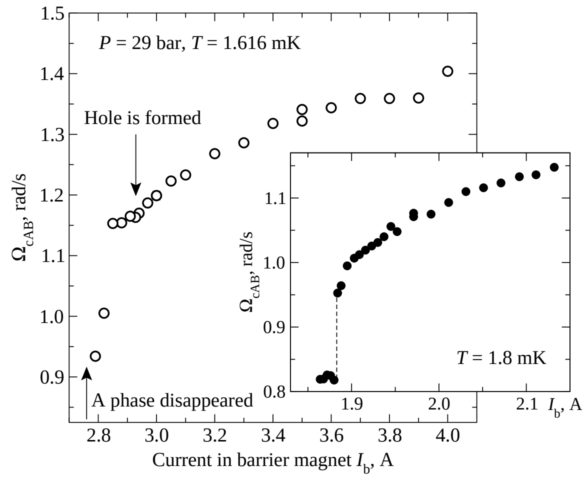

Fig. 2. Dependence of the AB-interface instability $\Omega_{\rm cAB}$ on the current $I_{\rm b}$ in the barrier magnet (at constant T, P). Measured in the direction of decreasing $I_{\rm b}$, $\Omega_{\rm cAB}$ is continuous when a ring-shaped AB interface with a hole is formed (Fig. 1, c→d). $\Omega_{\rm cAB}$ displays a sharp discontinuous jump down just before the AB interface finally disappears (Fig. 1f). *(Insert)* Discontinuity in $\Omega_{\rm cAB}$ measured with larger resolution in $I_{\rm b}$.

the A-phase volume. (b) Surface energy of the interface and at the sample boundary, $\sigma_{\rm AB}(S_{\rm AB} + S_{\rm AW} \cos\theta_{\rm AB})$, where $S_{\rm AB}$ and $S_{\rm AW}$ are the areas of the AB interface and of the container wall covered by the A phase, and $\theta_{\rm AB}$ is the contact angle of the AB interface with the wall. (c) Kinetic energy of the vortex-free superflow $\int_{\rm A} (1/2)(\rho_{\rm sA}(v_{\rm sA} - v_{\rm n})^2 - \rho_{\rm sB}(v_{\rm sB} - v_{\rm n})^2)\, dV$. Here $v_{\rm sA}$ and $v_{\rm sB}$ are generally very different since the two phases carry a different number of vortex lines. All contributions are expressed relative to the situation when the sample is filled only with B phase.

The parameters in the different energy terms are known or can be calculated [1]. The magnetic energy is the dominant contribution and thus the AB interface follows the profile $H_{\rm b} = H_{\rm AB}$ closely. Deviations due to surface tension and kinetic energy are limited to $0.1 - 0.2\,$mm (except when $H_{\rm AB} \to 0$). The numerical energy minimization in Fig. 1 shows how the AB interface moves up within the magnet (Fig. 1, a→b) when its current $I_{\rm b}$ is swept downward (or alternatively the sample is cooled). The solenoidal field is slightly larger at the outer boundary than at the axis of the sample. Thus with decreasing $I_{\rm b}$ the interface first becomes curved (Fig. 1b, c) and finally a hole is formed in the AB interface (Fig. 1d), *ie.* the stable A phase region is limited to a toroidal ring. This happens when the field $H_{\rm b}$ in the center of the magnet is reduced slightly below $H_{\rm AB}$ and the supercooled A phase in the top part of the sample becomes directly connected with the B phase in the bottom part. Its experimental signal is the A→B transition in the NMR spectrometer which monitors the top of the sample. With further decrease of $I_{\rm b}$ the A-phase ring shrinks towards the container wall (Fig. 1e, f) and finally disappears.

The critical velocity $\Omega_{\rm cAB}$ of the AB-interface instability is plotted in Fig. 2. The measured dependence follows $\Omega_{\rm cAB} \propto (\nabla H_{\rm b})^{1/4}$ [1], when $\nabla H_{\rm b}$ decreases with decreasing $I_{\rm b}$ as the AB boundary approaches the center of the barrier magnet. The dependence is continuous across the transition to the toroidal configuration of the A phase. This fact agrees with our understanding that the instability on the AB interface happens close to the outer sample boundary and is practically unaffected by changes which occur in the center.

When $I_{\rm b}$ is reduced further, a sharp downward jump is observed in $\Omega_{\rm cAB}$. The width of this discontinuity is $\Delta I_{\rm b}/I_{\rm b} < 10^{-3}$. At still lower currents $\Omega_{\rm cAB}$ remains approximately constant. If the sweep direction of $I_{\rm b}$ is reversed on this plateau then the dependence of $\Omega_{\rm cAB}$ on $I_{\rm b}$ shows good reversibility: The hysteretic shift in the location of the discontinuity in $\Omega_{\rm cAB}$ is $\Delta I_{\rm b}/I_{\rm b} < 10^{-2}$. At sufficiently small $I_{\rm b}$ the AB interface disappears which is marked by a discontinuous large increase in $\Omega_{\rm cB}$, now required to create B-phase vortices by other mechanisms [3]. If $I_{\rm b}$ is then swept back up, magnetic hysteresis in nucleating the A phase delays the formation of a new AB interface. This is marked by the return of the critical velocity to the $\Omega_{\rm cAB}$ curve in Fig. 2 at a value of $I_{\rm b}$ which is well above the jump in $\Omega_{\rm cAB}$.

In configurations at low $I_{\rm b}$ (Fig. 1f) the size of the interface is limited in the direction perpendicular to the flow to dimensions smaller than the wavelength of the instability in the infinite system ($\lambda \approx 0.7\,\mathrm{mm}$ in the present case). We have no calculations of the KH instability threshold for such a case. Naively one expects that such spatial restriction makes the interface stiffer and results in an increase of Ω_{cAB}. The opposite experimental observation remains so far unexplained.

The possibility to stabilize and control the shape of the AB interface allows new types of measurements on the interaction of topological defects with the phase boundary. For instance, what happens if the B-phase region within a toroidal A-phase ring is filled with vortex lines and one then starts to increase I_{b}? The B-phase vortex lines cannot escape and are squeezed together in the decreasing B-phase hole. One can then here study the interaction of the hard-core B-phase vortex lines with the approaching AB interface [4].

We thank R. Haley for discussions. This collaboration was carried out under the EU-IHP program (ULTI 3) and the ESF network COSLAB.

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