# Magnetization Plateaux in One-Dimensional Random Quantum Magnets

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**Abstract**

The effect of randomness on the magnetization plateaux in frustrated $S = 1/2$ Heisenberg chains is studied by the DMRG method. It is shown that the random mixture of two kinds of chains with plateaux due to spontanuous translational symmetry breakdown at $M_s/2$ ($M_s$: the full magnetization) does not show a plateaux at $M_s/2$. In the $S = 1/2$ frustrated Heisenberg chains with bond alternation and random sign strong bonds, the plateau at $M_s/2$ splits by randomness while an additional plateau is generated at $M = (1-p)M_s$.

*Key words:* random Heisenberg chain, bond alternation, frustration, DMRG, magnetization plateau

## 1. Introduction

The magnetization plateau in quantum spin systems is attracting much attention as the field induced spin gap states[1–3]. Oshikawa, Yamanaka and Affleck[4] proposed the necessary condition for the magnetization plateaux as $Q_{\rm GS}(S-M) =$ integer where $Q_{\rm GS}$ is the spatial periodicity of the magnetic ground state, $S$ is the magnitude of the spin and $M$ is the magnetization per site. It should be noted that $Q_{\rm GS}$ is not always equal to the spatial periodicity of the Hamiltonian $Q_{\rm Ham}$.

Recently, the randomness induced plateau is found in the $S = 1/2$ Heisenberg chains with bond alternation and random sign strong bonds[5]. Similar observation has been made by Cabra and coworkers[6] for random $q$-merized chains. On the other hand, Totsuka[7] discussed that the plateau in regular chains induced by the *imposed* periodicity ($Q_{\rm GS} = Q_{\rm Ham}$) is stable against weak randomness while the plateau induced by the spontanuous translational symmetry breakdown (STSB) ($Q_{\rm GS} \neq Q_{\rm Ham}$) is unstable against randomness due to Imry-Ma effect[8].

Therefore the randomness induces the magnetization plateau on one hand and on the other hand it destroys the plateau. In the present work, we investigate the interplay of these two apparently contradicting aspects of randomness effect on the magnetization plateau using the density matrix renormalization group (DMRG) method[9].

## 2. Model Hamiltonian

As a simplest model which exhibits a plateau accompanied by STSB, we consider the $S = 1/2$ Heisenberg chain with next-nearest-neighbour interaction whose Hamiltonian is given by,

$$H = \sum_{i=1}^{N-1} J_i \boldsymbol{S}_i \boldsymbol{S}_{i+1} + \sum_{i=1}^{N-2} J\delta \boldsymbol{S}_i \boldsymbol{S}_{i+2} \qquad (1)$$

where the bond alternation and randomness is introduced in the distribution of $J_i$. In the following, we consider the following two types of randomness: (i) random bond alternation: $J_i = J(1+(-1)^i\alpha_i)$ where $\alpha_i = \alpha_1(\alpha_2)$ with probability $p$ $(1-p)$ and (ii) random sign strong bonds: $J_{2i-1} = J > 0$ and $J_{2i} = J_{\rm A}(J_{\rm F})$

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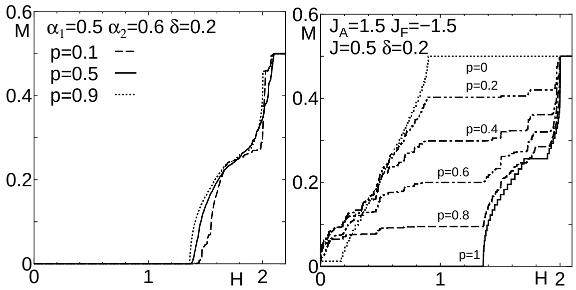


Fig. 1. (a) The magnetization curves of the random frustrated random Heisenberg chains with $\delta = 0.2, \alpha_1 = 0.5$ and $\alpha_2 = 0.6$. (b) The magnetization curves of the frustrated Heisenberg chains with bond alternation and random sign strong bonds with $J_{\rm A} = 1.5, J_{\rm F} = -1.5, J = 1.0$ and $\delta = 0.2$. The system size is $N = 82$ and average is taken over 40 samples.

with probability $p$ $(1-p)$ where $J_{\rm A} > J$ and $J_{\rm F} < 0$. In the pure case $(p = 1)$, this model has a plateau at magnetization $M = M_{\rm s}/2$ with STSB $(Q_{\rm GS} = 2Q_{\rm Ham})$ for appropriate values of $\alpha$ and $\delta$[2,3] where $M_{\rm s}$ is the saturation magnetization per site.

## 3. Numerical Results

Case (i) Frustrated Heisenberg chain with random bond alternation

For $\delta = 0.2$, the plateau region of the chains with $\alpha = 0.5$ and $\alpha = 0.6$ have finite overlap. Even in this overlap region, no plateau appears in the mixed chain as shown in Fig. 1(a). This verifies the fragility of the plateau accompanied by STSB against randomness due to Imry-Ma effect[8] as predicted by Totsuka[7].

It should be also remarked that the plateau with $M = 0$ (spin gap) remains stable. This plateau is due to the imposed bond alternation $(Q_{\rm GS} = Q_{\rm Ham})$ so that this result is also in accord with Totsuka's prediction[7].

Case (ii) Frustrated chains with bond alternation and random sign strong bonds

This model shows the randomness induced plateau at $M = M_{\rm s}(1-p)$ as in the unfrustrated case[5]. The fate of the plateau at $M = M_{\rm s}/2$ is different from that of case (i). The plateau no more exists at $M = M_{\rm s}/2$ for finite $p$ but it does not vanish. It splits into two plateaux with magnetizations $M_{\rm p1}$ and $M_{\rm p2}$ as depicted in Fig. 1(b). $M_{\rm p1}$ and $M_{\rm p2}$ depend nonlinearly on $1-p$ as shown in Fig. 2. This is in contrast to the case of the linear $p$-dependence of the plateau in random $q$-merized chain[6].

The $p$-dependence of $M_{\rm p1}$ and $M_{\rm p2}$ can be understood by introducing randomness into the strong coupling picture for the plateau state proposed by Totsuka[3,7]. The stable plateaux are expected at $M_{\rm p1} = M_{\rm s}[1-p+p^2/(1+p)]$ and $M_{\rm p2} = M_{\rm s}[1-p+p^3/(1+p)]$.

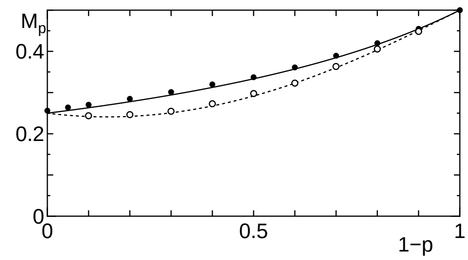


Fig. 2. The $(1-p)$-dependence of the magnetization on the plateau which reduces to the $M = M_{\rm s}/2$ plateau in the pure limit $(p = 1)$ for the frustrated Heisenberg chains with bond alternation and random sign strong bonds. The filled and open circles represent $M_{\rm p1}$ and $M_{\rm p2}$, respectively. The solid and dotted curevs represent the analytical estimation.

These are in agreement with the numerical results as plotted in Fig. 2 by solid and dotted curves.

## 4. Summary and Discussion

The magnetization plateaux in random quantum spin chains are investigated by the DMRG method. Up to now, the extensive experimental study of magnetization plateau with the interplay of randomness and frustration has not yet been carried out. Considering the variety of phenomena predicted analytically and numerically and recent progress of high magnetic field technique, fruitlful physics is expected in this field in the near future.

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