Dual Aharonov-Casher effect in singlet-exciton systems

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Abstract

Dual Aharonov-Casher (DAC) phase (or He-McKellar-Wilkens phase), which is defined as a quantum topological phase acquired by a *neutral* particle only with an *electric* dipole moment $\boldsymbol{\mu}_E$ being taken on a closed path around a *magentic* monopole wire, is theoretically investigated in singlet-exciton systems. In the Sangster's interference scheme, a moving $2s$ exciton in magnetic fields feels an effective electric field in co-moving frame, which causes a superposition of opposite parity states, i.e., $|2s\rangle$ and $|2p\rangle$, due to the *motional* Stark effect. The superposition gives rise to a nonvanishing electric dipole moment required for the DAC effect to the exciton. The accummulated phase is determined by detecting photon emissions from $2p$ states as a function of applied magnetic fields.

Key words: exciton; Aharonov-Casher effect; interference

1. Introduction

The Berry phase [1] leads to a wide variety of quantum-mechanical interference phenomena. The Aharonov-Casher (AC) phase [2] is one of such a phase based on a spin-orbit interaction and has been studied in various systems. In contrast, dual Aharonov-Casher (DAC) phase [3,4], which can be written in integral form as

$$\phi_{DAC} = -\frac{1}{\hbar c}\int [\boldsymbol{\mu}_E \times \boldsymbol{B}] \cdot d\boldsymbol{l}, \tag{1}$$

where $\boldsymbol{\mu}_E$ is an electric dipole moment and $\boldsymbol{B}$ is an applied magnetic field, has not attached interest because the movable system only with a permanent electric dipole moment does not exist. In this paper, we study DAC effect in single-exciton systems by means of the Sangster's scheme.

2. A singlet exciton with $\boldsymbol{\mu}_E$

An exciton is an electron-hole bound pair in a semiconductor, which can be regarded as a *neutral* composite Bose quasiparticle akin to the hydrogen atom. There are two possible exciton states depending on their spin states; *singlet* and triplet. The triplet exciton can be regarded as a neutral particle with spins so that it has been utilized for investigating the AC effect [5,6]. On the other hand, a singlet exciton cannot contribute to the AC effect because of no spins, but the DAC effect if it has $\boldsymbol{\mu}_E$. The clue as to how an exciton has $\boldsymbol{\mu}_E$ is being concealed in the exciton system itself [7]. The energy spectrum for exciton is described by the Rydberg energy, which depends only on the principal quantum number n, in addition to the kinetic energy associated with the center of mass,

$$E_{n,k} = \frac{-e^2}{2a_B n^2} + \frac{\hbar^2 k^2}{2(m_e + m_h)}, \tag{2}$$

where a_B is the hydrogenic Bohr radius and k is the exciton wavenumber. The excited hydrogenlike atoms have a degeneracy of states with opposite parity. Each state of the eigenstates has a definite parity and hence a zero dipole expectation value. However, the superposition of such states results in energy eigenstates without definite parity. Quite generally, for an energy state that we can write as a superposition of opposite parity states, it is permissible to have a nonvanishing permanent electric dipole moment.

3. $\boldsymbol{\mu}_E$ induced by *motional* Stark effect

Consider a particle with a velocity v moving in a magnetic field. The particle in the co-moving frame feels an effective electric field $\boldsymbol{E}' = -\boldsymbol{v} \times \boldsymbol{B}/c$ with c being the speed of light in the small velocity limit, even if the particle is *neutral* and $\boldsymbol{E} = 0$ in the laboratory frame. This is due to Lorentz transformation for the electric and magnetic fields in the theory of relativity. The particle can interact with the effective electric field if it has an electric dipole moment $\boldsymbol{\mu}_E$. The dipole interaction is described by

$$\hat{H}_{dipole} = -\boldsymbol{\mu}_E \cdot \boldsymbol{E}', \tag{3}$$

in the co-moving frame of the electric dipole. In the DAC laboratory frame, it can be rewritten as

$$\hat{H}_{DAC} = \boldsymbol{\mu}_E \cdot (\boldsymbol{v} \times \boldsymbol{B}/c), \tag{4}$$

which is so-called Röntgen interaction [4]. Under the DAC Hamiltonian $\hat{H}_{DAC}$, the state evolves as

$$|\Psi(t)\rangle = \exp(-i\hat{H}_{DAC}t/\hbar)|\Psi(0)\rangle. \tag{5}$$

Now suppose that an exciton is prepared in the first-excited $2s$ state. Due to the effective electric field, the degenerate $m = 0$ state splits into two components, with energy eigenvalues and eigenfunctions given by $\Delta^{\pm} = \pm 3a_B eE'$ and $|\psi^{\pm}\rangle = (|2s, m = 0\rangle \pm |2p, m = 0\rangle)/\sqrt{2}$, respectively. The $2s$ exciton thus evolves into supeposition of $2s$ and $2p$ states due to the *motional* Stark effect,

$$|\Psi(t)\rangle = \cos(\phi_{DAC}t/t_0)|2s\rangle + i\sin(\phi_{DAC}t/t_0)|2p\rangle, \tag{6}$$

$$\phi_{DAC} = 3a_B eBL/\hbar c = 3a_B eBvt_0/\hbar c, \tag{7}$$

while the exciton travels a length L under a magnetic field. In new basis the states $|\psi^{\pm}\rangle = (|2s, m = 0\rangle \pm |2p, m = 0\rangle)/\sqrt{2}$ are not parity eigenstates and have definite *up* or *down* dipole value required for the DAC effect. The notable features of the DAC phase are velocity independence and proportionality to the applied magnetic field.

4. DAC effect by the Sangster's scheme

The ordinary scheme for observing the AC phase involves two coherent beams with the same magnetic moment traveling on opposite sides of a charged wire. However it is not nessesary for the path to enclose a charged wire in order to observe the AC effect. In the Sangster's scheme, the two coherent beams have opposite magnetic moments and are not spatially separated: they pass through the same electric field [8]. In this way, the motional Stark effect is also responsible for the requirement of the Sangster's scheme, which is a superposition of dipoles *in one exciton on one path*.

The $2p$ state decays rapidly to the ground state by a dipole transition and has a very short lifetime. This can be estimated by the oscillator strength f_n from np excitonic state to ground state given by

$$f_n = \frac{2V\hbar|M|^2}{3m\omega}\frac{(n^2-1)}{\pi a_B^5 n^5}, \tag{8}$$

where V is the volume of the sample and M is a dipole transition matrix element. The accummulated phase is therefore determined by detecting photon emissions from $2p$ states as a function of applied magnetic fields.

A possible candidate for a DAC experiment is Cu_2O which is a forbidden direct-gap semiconductor, resulting in relatively long lifetimes [9]. Hence this material has predominantly used in studies of Bose-Einstein condensation of elementary excitations. The excitonic Bohr radius is ten times larger than the hydrogenic Bohr radius. This makes the DAC phase larger and the observation easier. This is an advantage of excitonic systems. For example, if B is measured in Gauss and L in centimeters, then Eq. (7) can be written $\phi_{DAC} = 1.5BL$. This relation tells us that the detection of the phase might be accessible to the experiments.

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