Josephson π states in superfluid ^{3}He B-phase/A-phase/B-phase junctions

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Abstract

The $\hat{\boldsymbol{l}}$ texture in the A phase of superfluid ^{3}He B-phase/A-phase/B-phase hybrid junction was a key to form the π state with higher critical current observed in Berkeley. Here we investigate the effect of $\hat{\boldsymbol{n}}$ texture in the B phase on current-phase relations for the BAB junction with a fixed $\hat{\boldsymbol{l}}$ texture. We show that the $\hat{\boldsymbol{n}}$ texture greatly changes the current-phase relation just like $\hat{\boldsymbol{l}}$ texture does. The change is due to the $\hat{\boldsymbol{n}}$-texture-induced modification of the spin structure in the A phase of the BAB junction via the AB boundary condition. As a result, current-phase relations in BAB junctions are determined by the $\hat{\boldsymbol{d}}$ vector in the A phase depending on both $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{l}}$ textures.

Key words: Josephson effect; superfluidity; helium 3

Bistable π states in superfluid ^{3}He weak links (**L** state with lower critical current and **H** state with higher critical current) have been observed by Marchenkov *et al.* in Berkeley [1]. In order to explain these π states, several theories have been proposed [2,3]. In the previous paper [4], we studied the current-phase relations for B-phase/A-phase/B-phase (BAB) hybrid Josephson junctions, which might be possible to model weak links in superfluid ^{3}He. We succeeded in obtaining the current-phase relations which had major features of **H** state, and proposed a new mechanism of π-state formation due to $\hat{\boldsymbol{l}}$ texture in the A phase ($\hat{\boldsymbol{l}}$-texture mechanism). However, the contribution to the current-phase relations from the $\hat{\boldsymbol{n}}$ texture in the B phase remains unclear. In this paper, we investigate the effect of $\hat{\boldsymbol{n}}$ textures on the current-phase relations for the BAB junctions.

The order parameter of the A phase is defined by a triad $(\hat{\boldsymbol{w}}_1, \hat{\boldsymbol{w}}_2, \hat{\boldsymbol{l}})$ in orbital space and a vector $\hat{\boldsymbol{d}}$ in spin space. For the B phase we need a rotational matrix $R(\hat{\boldsymbol{n}}, \theta)$ with rotational axis $\hat{\boldsymbol{n}}$ and rotational angle θ. We modify the previous configuration so that the $\hat{\boldsymbol{n}}$ vector in the left-side B phase ($\hat{\boldsymbol{n}}^l$) might tilt from the normal of the AB interface by an angle β as shown in the inset of Fig. 1. We can get back to the previous configuration in [4] when $\beta = \pi$. The $\hat{\boldsymbol{l}}$ vectors in two A-phase regions ($\hat{\boldsymbol{l}}^l$ and $\hat{\boldsymbol{l}}^r$) are assumed to be tilted from the AB interface by the angle $\alpha = 0.2\pi$ as we considered in [4]. We also assume that $\hat{\boldsymbol{l}}^l$, $\hat{\boldsymbol{l}}^r$, $\hat{\boldsymbol{n}}^l$, and $\hat{\boldsymbol{n}}^r$ are in the same plane for brevity.

Figure 1 shows the current-phase relations for different values of β calculated by the supercurrent formula derived in the previous paper. The current-phase relations change drastically as β varies. There are two states defined by the zeros of current with positive gradients, i.e., "0 state" and "π state", at 0 and π phase differences. When $\beta = 1.1\pi$, the 0 state is the equilibrium state as in a usual junction ("0 junction"). While, the π state becomes more stable than the 0 state when $\beta = 0.7\pi$. In this case, a so-called "π junction" is realized. A crossover between "0 junction" and "π junction" occurs as a function of β.

In order to clarify the dependence of current-phase relations on $\hat{\boldsymbol{n}}$ textures, we plot the phase differences at the peak positions of current as shown in Fig. 2. The β dependence of peak positions for BAB junctions (solid

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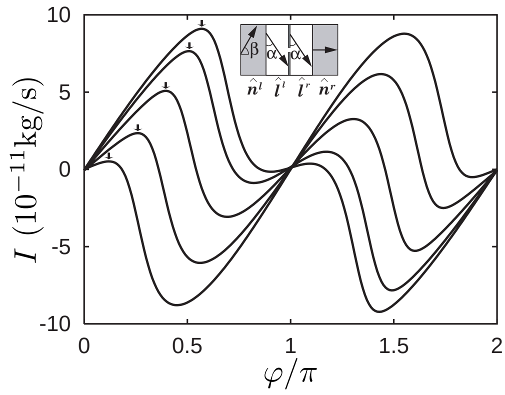

Fig. 1. Current-phase relations for $\beta = 0.7\pi$, 0.8π, 0.9π, 1.0π, and 1.1π in increasing order of gradient at the origin. The currents were calculated by using the parameters of the Berkeley experiments at $0.1T_\mathrm{c}$ with T_c being the critical temperature for the B phase. The arrows indicate the peak positions. The inset shows a schematic diagram of a BAB junction.

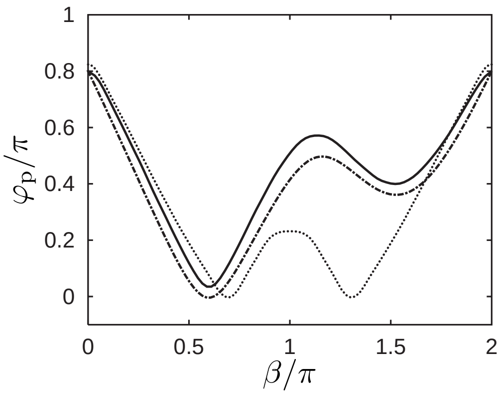

Fig. 2. β dependence of peak positions for BAB junction (solid line) and BB junction (dotted line). The dashed-dotted line shows the shift calculated by Eq. (1) with the offset 0.8π which corresponds to the peak position at $\beta = 0$ and depends on the temperature.

line) is asymmetric with respect to $\beta = \pi$, $\varphi_\mathrm{p}(2\pi - \beta) \neq \varphi_\mathrm{p}(\beta)$, and differs considerably from that for BB junctions (dotted line) around $0.5\pi \lesssim \beta \lesssim 1.7\pi$.

Now let us discuss the mechanism why the $\hat{\boldsymbol{n}}$ texture influences on current-phase relations. In the BAB junction, $\hat{\boldsymbol{n}}$ in the B phase and $\hat{\boldsymbol{d}}$ in the A phase are connected by the boundary conditions at the AB interface. The boundary conditions at AB interfaces are determined by minimizing the surface energy [5–7]. The most important condition is $\hat{\boldsymbol{d}} = R(\hat{\boldsymbol{n}}, \theta_\mathrm{L})\hat{\boldsymbol{w}}_1$, where $\theta_\mathrm{L} = \cos^{-1}(-1/4)$ (Leggett angle). The difference of the $\hat{\boldsymbol{n}}$ vectors in two B-phase regions results in the difference of the $\hat{\boldsymbol{d}}$ vectors ($\hat{\boldsymbol{d}}^l$ and $\hat{\boldsymbol{d}}^r$) in the A-phase regions. If we take $\hat{\boldsymbol{d}}^l \times \hat{\boldsymbol{d}}^r$ as the spin-quantization axis, quasiparticles feel phase differences $\varphi \mp \cos^{-1}(\hat{\boldsymbol{d}}^l \cdot \hat{\boldsymbol{d}}^r)$ ($-$ for spin $\uparrow$, $+$ for spin $\downarrow$) [2,4]. The current-phase relations of spin $\uparrow$ and $\downarrow$ shift by $\delta\varphi = \cos^{-1}(\hat{\boldsymbol{d}}^l \cdot \hat{\boldsymbol{d}}^r)$ and $-\delta\varphi$, respectively. Therefore, the peak position of the total current should shift by $-\delta\varphi$. The excess phase can be expressed as

$$\begin{aligned}
\delta\varphi &= \cos^{-1}(\hat{\boldsymbol{d}}^l \cdot \hat{\boldsymbol{d}}^r), \\
\hat{\boldsymbol{d}}^l \cdot \hat{\boldsymbol{d}}^r &= \frac{1}{16}\{6 + 10\cos(2\beta) \\
&\quad +15(1-\cos\beta)\sin\alpha\sin(\alpha-\beta)\}. \quad (1)
\end{aligned}$$

In the case of BB junction, the peak position cannot be expressed in such a compact form because the $\hat{\boldsymbol{d}}$ vector depends on the quasiparticle momentum. The dashed-dotted line in Fig. 2 shows the excess phase calculated by Eq. (1). The behavior of this excess phase is consistent with the behavior of the peak position for the BAB junction. The asymmetry in the β dependence of peak positions for BAB junctions comes from the last term of the right hand side in Eq. (1). The asymmetry is controlled by the angle α which determines the direction of the $\hat{\boldsymbol{l}}$ vector. Therefore, the direction of the $\hat{\boldsymbol{l}}$ vector is deduced from the current-phase relations under different configurations of $\hat{\boldsymbol{n}}$ textures.

In conclusion, we have confirmed that the origin of the $\hat{\boldsymbol{n}}$-texture dependence of current-phase relations for BAB junctions is the excess phase due to the difference between $\hat{\boldsymbol{d}}^l$ and $\hat{\boldsymbol{d}}^r$ which is produced by the $\hat{\boldsymbol{n}}$ texture via the AB boundary condition. Therefore, current-phase relations in BAB junctions are determined by the $\hat{\boldsymbol{d}}$ vector in the A phase depending on both $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{l}}$ textures. Since the direction of $\hat{\boldsymbol{n}}$ in the B phase can be modified by applying magnetic fields, the information of the A phase – (i) the existence of the A phase and (ii) the direction of $\hat{\boldsymbol{l}}$ – can be obtained from the systematic analysis of future experiments under magnetic fields.

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