Microscopic study of low-κ type-II superconductors

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Abstract

Thermodynamics and vortex lattices of low-κ type-II superconductors are studied numerically within the Eilenberger equations of superconductivity. Below some critical value $\kappa_c(T)$ of the Ginzburg-Landau parameter $\kappa = \lambda/\xi$, there is a first order transition at H_{c1} (type-IIa superconductivity) as a consequence of attractive vortex-vortex interaction. For $\kappa > \kappa_c$ phase transition is of the second order (type-IIb superconductivity). Phase boundary $\kappa_c(T)$ that separates type-IIa and type-IIb superconductivity has been calculated.

Key words: type-II superconductivity; low-κ superconductors; vortex-vortex interaction

Superconductors are divided into two groups, type-I and type-II, based on their behavior in magnetic field. It can be shown within Ginzburg-Landau (GL) approach that the difference between two types of superconductivity originates from the sign of vortex-vortex interaction, which can be attractive or repulsive. Sign of the interaction is controlled by GL parameter $\kappa = \lambda/\xi$, ratio of penetration depth and coherence length near T_c. In type-I superconductors ($\kappa < 1/\sqrt{2}$) vortices attract each other and the phase transition from Meissner to normal state is first order. In type-II superconductors ($\kappa > 1/\sqrt{2}$), vortex-vortex interaction is repulsive and the phase transition from Meissner to the mixed state at H_{c1} is second order. It has been verified experimentally since a long time that even in type-II superconductors with $\kappa \sim 1$ interaction among vortices can be attractive (for a review of experimental results see Ref. [1]). Vortex attraction is manifested as a first order phase transition from Meissner to the mixed state. This type of superconductivity we denote as type-IIa to distinguish it from the type-IIb superconductivity with purely repulsive vortex-vortex interaction. Most of the theoretical efforts to understand interaction of vortices as a function of parameter κ relied on the extended GL model. Utilization of Bogomolnyi method to treat GL equations for κ close to $1/\sqrt{2}$ gave the further impetus for this research direction [1,2]. However, range of applicability of GL formalism is confined to the region near T_c. To calculate the full temperature dependence of $\kappa_c(T)$, boundary between type-IIa and type-IIb superconductivity, one should rather turn to the microscopic theory of superconductivity. The only attempt to calculate numerically $\kappa_c(T)$ without approximations besides those immanent to numerical procedure is due to Klein [3].

The purpose of this work is to shed more light on the problem of vortex attraction. Phase diagram (κ, T) of type-I, type-IIa and type-IIb superconductivity has been calculated numerically. We consider clean superconductors with isotropic cylindrical Fermi surface, choice motivated mainly by the speed of calculation. The quantitative results will of course be different from the 3D case but the qualitative (κ, T) phase diagram will be quite similar.

For the isotropic gap Eilenberger equations are [4]

$$[\omega + \mathbf{u}\,(\nabla + \mathrm{i}\mathbf{A})]\, f = \Psi g, \tag{1}$$

$$[\omega - \mathbf{u}\,(\nabla - \mathrm{i}\mathbf{A})]\, f^{\dagger} = \Psi^{*} g. \tag{2}$$

These are supplemented by the self-consistency equations for the gap function Ψ and vector-potential $\mathbf{A}$

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$$\Psi \ln t = 2t \sum_{\omega>0} \left[\langle f \rangle - \frac{\Psi}{\omega} \right], \tag{3}$$

$$\nabla \times \nabla \times \mathbf{A} = -\frac{2t}{\widetilde{\kappa}^2} \operatorname{Im} \sum_{\omega>0} \langle \mathbf{u} g \rangle \,. \tag{4}$$

Equations are written in Eilenberger dimensionless units. Here, ω is Matsubara frequency, $t = T/T_c$, parameter $\tilde{\kappa}^2 = 7\zeta(3)\kappa^2/8$, $\mathbf{u}$ is Fermi velocity direction and f, $f^\dagger$, g are Eilenberger functions, $g^2 + ff^\dagger = 1$. Expression for the free energy density difference between superconducting and normal state $F = F_s - F_n$ is given by (simplified with the help of Eilenberger and self-consistency equations)

$$F = \widetilde{\kappa}^2 \overline{(\nabla \times \mathbf{A})^2} - t \sum_{\omega>0} \left\langle \overline{\frac{1-g}{1+g} (\Psi^* f + \Psi f^\dagger)} \right\rangle. \tag{5}$$

Notation for spatial average $\overline{C} = (\overline{B}/2\pi) \int_{cell} C dS$ is used, where $\overline{B}$ is magnetic induction. Traditionally Eilenberger equations are solved numerically by the so-called "explosion" method (see Ref. [5]). Here another one approach which takes the advantage of periodicity of the vortex lattice has been adopted. Instead in real space, we solved equations numerically in Fourier space (details of the method will be published elsewhere).

Boundary between type-I and type-II superconductivity can be obtained from the condition $H_c = H_{c2}$. Thermodynamic critical field H_c is defined via the zero-field energy difference between normal and superconducting state

$$F_n - F_s = 2t \sum_{\omega>0} \frac{\Psi^4}{\sqrt{\omega^2 + \Psi^2} \left(\omega + \sqrt{\omega^2 + \Psi^2} \right)^2}, \tag{6}$$

where $\Psi(t)$ is zero-field gap. Thermodynamic critical field is given by $H_c^2(\widetilde{\kappa}, t) = \Delta F(t)/\widetilde{\kappa}^2$. Therefore critical value for the GL parameter κ is defined as (2D case)

$$\kappa_c(t) = \frac{1}{H_{c2}^{2D}} \sqrt{\frac{8}{7\zeta(3)} (F_n - F_s)}. \tag{7}$$

To find the boundary between type-IIa and type-IIb superconductivity we proceeded as following. For a particular value of GL parameter κ and temperature t, a set of Eilenberger equations is solved numerically for various values of magnetic induction $\overline{B}$, so we can construct Helmholtz free energy density $F(\overline{B})$. For a given applied filed H proper thermodynamic potential is Gibbs energy density $G(\overline{\mathbf{B}})$ which is at minimum when the thermodynamic equilibrium is achieved, $\partial G/\partial \overline{B} = 0$. This condition is rewritten via $F(\overline{B})$

$$H(\overline{B}) = \frac{1}{2\widetilde{\kappa}^2} \frac{\partial F}{\partial \overline{B}}. \tag{8}$$

If there are meta-stable states, as we expect to be in type-IIa superconductors, function $\overline{B}(H)$ is multi-valued and to find a true magnetic induction one should look for global minimum of Gibbs energy $G(\overline{B}) = F(\overline{B}) - 2\tilde{\kappa}^2 \overline{B} H$. Therefore from the behavior of Gibbs energy we can reconstruct the $\overline{B}(H)$ curve. In type-IIa superconductors from $H = 0$ up to $H = H_{c1}^*$ minimum Gibbs energy is for $\overline{B} = 0$, Meissner state. At $H = H_{c1}^*$ (which is less than $H_{c1} = (1/2\widetilde{\kappa}^2) \partial F/\partial \overline{B}|_{\overline{B}=0}$), minimum energy is at $\overline{B} = B_0$, first order phase transition from the Meissner state to the mixed state (jump in magnetization). It means that the energy of well separated vortices $\overline{B} \approx 0$ is higher than vortex-lattice energy at $\overline{B} = B_0$, due to attraction of vortices. For a fixed temperature the value of induction jump B_0 decreases with increasing κ and at some critical value κ_c, $B_0 = 0$. Then for all $\kappa > \kappa_c$ we have a second order phase transition from the Meissner to mixed state, i.e. type-IIb superconductivity.

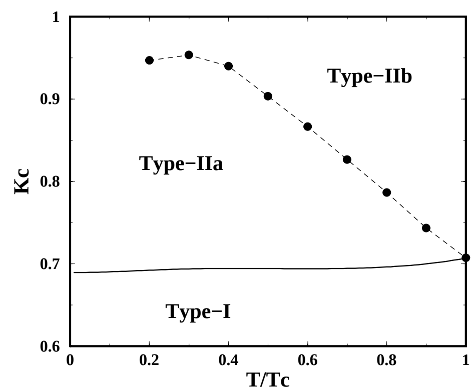

Fig. 1. Phase boundary between type-I, type-IIa and type-IIb superconductivity for isotropic 2D Fermi surface.

Calculations are performed in temperature interval $t = 0.2 - 0.9$ to find the critical parameter κ_c that separates type-IIa and type-IIb superconductivity. The results are shown on Fig. 1. Boundary line is qualitatively similar to the 3D case obtained by different numerical scheme (explosion method). We also checked that the hexagonal vortex-lattice has lower energy than the square one.

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