

Field dependence of vortex structure in $p_x \pm ip_y$ -wave superconductors

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Abstract

To understand the vortex states in Sr_2RuO_4 , we investigate differences of the vortex structure for two chiral pairings $p_x \pm ip_y$, using quasiclassical Eilenberger theory and the Ginzburg-Landau theory. The induced opposite chiral component of the pair potential plays an important role in the vortex structure. It produces an \sqrt{H} -behavior of the zero-energy density of states at higher field. We also consider the anisotropic case of the Fermi surface and superconducting gap.

Key words: vortex structure; p -wave superconductor; local density of states; quasiclassical theory

For the superconducting state in quasi-two-dimensional metal Sr_2RuO_4 , the pairing symmetry is suggested to be the chiral p -wave pairing with the basic form $p_{\pm} \sim p_x \pm ip_y$ and inplane equal-spin pairing [1,2]. This degeneracy is lifted under external magnetic field perpendicular to the basal plane, since p_{\pm} is a broken time reversal symmetry state with an orbital angular momentum along the z -axis. Then, the vortex in the mixed state shows the different structure for the p_+ -wave and the p_- -wave superconductivity [3,4]. We analyze the magnetic field (H) dependence of the vortex structure for the chiral states, based on the quasiclassical Eilenberger theory and the Ginzburg-Landau (GL) theory.

In the calculation of the quasi-classical theory, we study the pair potential, the internal field, the local density of states (LDOS), and the free energy in the vortex lattice state, numerically solving the Eilenberger equations

$$\left\{ \omega_n + \frac{i}{2} \mathbf{v}_F \cdot \left(\frac{\nabla}{i} + \frac{2\pi}{\phi_0} \mathbf{A}(\mathbf{r}) \right) \right\} f(i\omega_n, \mathbf{k}_F, \mathbf{r}) = \Delta(\mathbf{k}_F, \mathbf{r}) g(i\omega_n, \mathbf{k}_F, \mathbf{r}), \quad (1)$$

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$$\left\{ \omega_n - \frac{i}{2} \mathbf{v}_F \cdot \left(\frac{\nabla}{i} - \frac{2\pi}{\phi_0} \mathbf{A}(\mathbf{r}) \right) \right\} f^\dagger(i\omega_n, \mathbf{k}_F, \mathbf{r}) = \Delta^*(\mathbf{k}_F, \mathbf{r}) g(i\omega_n, \mathbf{k}_F, \mathbf{r}), \quad (2)$$

where $g(i\omega_n, \mathbf{k}_F, \mathbf{r}) = [1 - f(i\omega_n, \mathbf{k}_F, \mathbf{r}) f^\dagger(i\omega_n, \mathbf{k}_F, \mathbf{r})]^{1/2}$ with the Matsubara frequency ω_n , the Fermi velocity $\mathbf{v}_F = \partial\epsilon(\mathbf{k})/\partial\mathbf{k}$, and the flux quantum ϕ_0 . The relative momentum and the center of mass coordinate of the Cooper pair are, respectively, indicated by \mathbf{k}_F on the Fermi surface and by \mathbf{r} . The pair potential $\Delta(\mathbf{k}_F, \mathbf{r})$ and the vector potential $\mathbf{A}(\mathbf{r})$ are self-consistently calculated from the quasiclassical Green's functions $g(i\omega_n, \mathbf{k}_F, \mathbf{r})$, $f(i\omega_n, \mathbf{k}_F, \mathbf{r})$ and $f^\dagger(i\omega_n, \mathbf{k}_F, \mathbf{r})$. We set the energy cutoff $\omega_c = 20T_c$ and the GL parameter $\kappa_{\text{BCS}} = 2.7$. In the following, lengths and magnetic fields are measured in units of $\xi_0 = v_F/\Delta_0 = \pi\xi_{\text{BCS}}$ (ξ_{BCS} is the BCS coherence length) and ϕ_0/ξ_0^2 , respectively. The LDOS is given by $N(E, \mathbf{r}) = \langle \text{Im}g(i\omega_n \rightarrow E + i\eta, \mathbf{k}_F, \mathbf{r}) \rangle_{\mathbf{k}_F}$ with the Fermi surface average $\langle \dots \rangle_{\mathbf{k}_F}$. In the chiral p -wave pairing, the pair potential and the pairing interaction are set as

$$\Delta(\mathbf{k}_F, \mathbf{r}) = \Delta_+(\mathbf{r})\phi_+(\mathbf{k}_F) + \Delta_-(\mathbf{r})\phi_-(\mathbf{k}_F), \quad (3)$$

$$V(\mathbf{k}'_F, \mathbf{k}_F) = \bar{V}[\phi_+^*(\mathbf{k}'_F)\phi_+(\mathbf{k}_F) + \phi_-^*(\mathbf{k}'_F)\phi_-(\mathbf{k}_F)] \quad (4)$$

with the pairing functions $\phi_{\pm}(\mathbf{k}_F)$ for the p_{\pm} -wave

components. Unless we consider the induced component, there are no differences for the p_+ -wave and the p_- -wave cases, because the formulation of the Eilenberger theory and the results of the vortex structure are reduced to the same one as in the s -wave case with a pairing function $|\phi_{\pm}(\mathbf{k}_F)|$. Therefore, two component pair potential is intrinsic and essential for the vortex structure in the chiral p -wave superconductors. We also calculate the vortex structure in the s -wave case as a reference.

The vortex structure in the isotropic case of a cylindrical Fermi surface $\mathbf{k}_F = k_F(\cos\theta, \sin\theta)$ and $\phi_{\pm}(\mathbf{k}_F) = e^{\pm i\theta}$ was reported in Ref. [4]. The differences of the p_{\pm} -wave cases come from the structure of the induced opposite chiral component of the pair potential. The phase winding structure of the induced component is different depending on the chirality. The phase of the induced $\Delta_-(\mathbf{r})$ ($\Delta_+(\mathbf{r})$) in the p_+ -wave (p_- -wave) state has +3-winding (-1-winding) at the vortex center, where the dominant pair potential has +1-winding. In the p_+ -wave case, the amplitude of the induced p_- -wave component is small and reduced to zero near H_{c2} . Then, the vortex structure is similar to that of the s -wave case, and H_{c2} is same as in the s -wave case in the two-dimensional Fermi surface. In the p_- -wave case, the induced p_+ -wave component is large. The superconductivity can survive up to a high field, giving high H_{c2} . The induced component also produces an anomalous internal field distribution. When we compare the free energy, the p_- -wave state is stable, and the p_+ -wave state is metastable.

The two-component GL equations for $\Delta_+(\mathbf{r})$ and $\Delta_-(\mathbf{r})$ are derived from the microscopic theory. We also calculate the vortex structure from the GL equations, and obtain qualitatively the same vortex structure. Further, we find the transition from the p_+ -wave state to the p_- -wave state, because the p_- -wave state has a lower free energy. The domain wall between the p_{\pm} -wave states moves, and the area of the p_+ -wave domain shrinks with increasing field.

Next, to study the anisotropy effect, we consider the case of Fermi surface given by the dispersion $\epsilon(\mathbf{k}) = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y$ with $t' = 0.5t$ and the chemical potential $\mu = 1.65t$. This reproduces the Fermi surface shape of γ -sheet in Sr_2RuO_4 [3]. The pairing functions are defined as $\phi_{\pm}(\mathbf{k}_F) = \sin k_x \mp i \sin k_y$. By this definition, the p_{\pm} -wave states have the same phase winding as in the isotropic case noted above. In this anisotropic case, we show the zero-energy LDOS $N(E = 0, \mathbf{r})$ within a unit cell of the vortex lattice in Fig. 1(a). We consider the square vortex lattice where the nearest neighbor vortex is located in the 45° -direction from the a -axis, following the observed vortex lattice configuration [5]. The magnitude of the zero-energy states has a peak at the vortex core, and extends to the a - and b -axis directions (diagonal di-

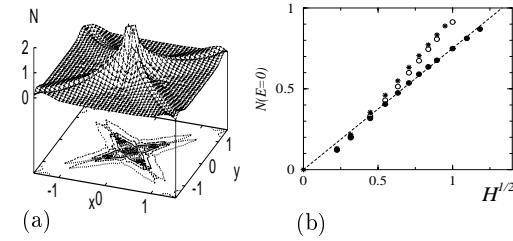


Fig. 1. (a) Zero-energy LDOS $N(E = 0, \mathbf{r})$ within a square unit-cell of the vortex lattice. Vortex is located at the center of the figure. (b) Zero-energy DOS $N(0)$ as a function of \sqrt{H} for the p_- -wave (•) and the p_+ -wave (○) cases. Points * show the s -wave case with a pairing function $|\phi_{\pm}(\mathbf{k}_F)|$. $N(0)$ is normalized by the normal state DOS. Dashed lines show the relation $N(0) \propto \sqrt{H}$.

rections of the unit cell in the Fig. 1(a)) because the \mathbf{k}_F -resolved coherence length $\xi(\mathbf{k}_F) = v_F(\mathbf{k}_F)/|\phi(\mathbf{k}_F)|$ is longer in these directions [3].

When we quantitatively consider the field dependence of the zero energy DOS $N(0)$, averaging the LDOS, we see the effect of the chiral p -wave superconductivity. The stable p_- -wave case shows an \sqrt{H} -behavior at higher field, as displayed in Fig. 1(b). It is because the energy gap $|\Delta(\mathbf{k}_F, \mathbf{r})|$ can be small in some \mathbf{k}_F -directions even outside of a vortex core, when the induced component $\Delta_+(\mathbf{r})$ is large in Eq. (3) in addition to the dominant $\Delta_-(\mathbf{r})$. Since the induced component is restricted within a vortex core region, $N(0)$ deviates from an \sqrt{H} -relation at low field. This type H -dependence of the DOS is observed in the specific heat measurement[6].

In summary, we have studied the vortex structure in $p_x \pm ip_y$ -wave superconductors, using the quasi-classical Eilenberger theory and the GL theory. By the effect of the induced opposite chiral component, the H -dependence of the DOS shows an \sqrt{H} -relation. The LDOS around the vortex core extends toward the direction with the longer \mathbf{k} -resolved coherence length.

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