Superconducting and density-wave correlation functions in carbon nanotubes

Kenji Kamide [1], Munehiro Nishida, Susumu Kurihara

Department of Physics, Waseda University, Okubo, Shinjuku-ku, Tokyo 169-8555, Japan

Abstract

We discuss the phase diagrams obtained by calculating temperature dependences of superconducting and density-wave correlation functions for a single $(5,0)$ carbon nanotube (CN). We use one-loop renormalization group method within logarithmic accuracy. In this system we must specify scattering channels in terms of momentum along the circumferential direction as well as the axis direction, because $(5,0)$ CN has two degenerate bands crossing the Fermi energy with circumferential momenta. We find that the most divergent order is singlet superconducting or charge-density wave with 0 or $\frac{5\pi\hbar}{2\pi R}$ circumferential momentum, where R is the tube radius.

Key words: carbon nanotube; superconductivity; density-wave; renormalization group

The discovery of carbon nanotubes (CN) in 1991 [1] have attracted much attention because of its potential for new physics, as well as applications. Recently, Tang *et al.* [2] reported evidence for superconductivity in (5,0)CN systems, where the tubes are coupled to each other very weakly. In this paper, we study the low energy properties of a quasi-one-dimensional electron system with three bands as a model for (5,0)CN, using one-loop renormalization group method.

(5,0)CN has three bands crossing the Fermi energy [3]. One ($\gamma = 1$) is nondegenerate but the others ($\gamma = 2,3$) are doubly degenerate with opposite momenta in circumferential direction. Since we are concerned with low energy extations, we neglect the effect of bands which do not cross the Fermi energy. We take into account forward, backward, and Umklapp scatterings near the Fermi level. Since the two bands with circumferential momenta cross the Fermi energy, we shall pay attention to the fact that the scattering amplitudes depend on the momentum transfer in the circumferential direction as well as the axis direction.

We have derived scaling equations for these couplings and correlation functions for all possible phases [4], i.e. charge-density wave (CDW), spin-density wave (SDW), singlet superconductor (SS) and triplet superconductor (TS), following the method of Solyom [5]. The phase diagrams are obtained by solving these scaling equations numerically.

We can divide this system into three independent groups by considering the types of scattering processes. The expression (γ, γ') below denotes scatterings between electrons in branch γ and γ'.

Group 1 : $(\gamma, \gamma') = (1,1)$, $(1,2)$ or $(1,3)$

These channels have only two independent couplings $b_{\gamma,\gamma'}$ and $f_{\gamma,\gamma'}$, corresponding to backward and forward scattering in the axis direction. Backward scatterings in $(1,2)$ and $(1,3)$ changes circumferential momentum as well. We can discuss this group in the same way as the one dimensional system investigated by Solyom [5]. The phase boundary is determined by $b_{\gamma,\gamma'} = 0$ or $b_{\gamma,\gamma'} - 2f_{\gamma,\gamma'} = 0$. We define gapful region as the region where the couplings are scaled to be divergent, otherwise gapless region. The $b_{\gamma,\gamma'} < 0$ domain corresponds to gapful phase, and $b_{\gamma,\gamma'} > 0$ corresponds to the gapless phase. In the gapful phase, CDW dominates in the $b_{\gamma,\gamma'} - 2f_{\gamma,\gamma'} < 0$ domain, and SS dominates in the $b_{\gamma,\gamma'} - 2f_{\gamma,\gamma'} > 0$ domain.

[1] E-mail: kamide@kh.phys.waseda.ac.jp

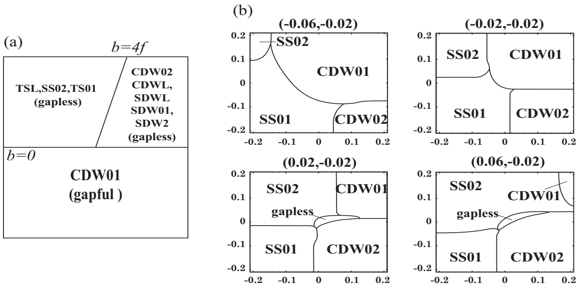

Fig. 1. Phase diagram for group 2. The value of (a) (b, f), (b) (g_2^1, g_1^1), normalized by $2\pi v_F$ are taken at the horizontal and vertical axis respectively. Values of (b, f) are shown above the diagrams in (b).

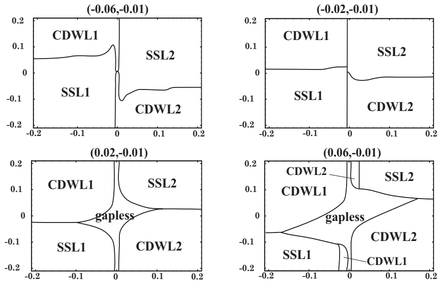

Fig. 2. Phase diagram for group 3. The value of $(\frac{g_2^3}{2\pi v}, \frac{g_1^3}{2\pi v})$ are taken at the horizontal and vertical axis respectively, where $v \equiv (v_{F,2}+v_{F,3})/2$. The couplings are taken to be $g_1^1 = g_1^4 \equiv b$, $g_2^2 = g_2^4 \equiv f$ and are shown above the diagrams.

Group 2 : $(\gamma, \gamma') = (2, 2)$, or $(3, 3)$

These channels have six independent couplings $\{g_1^1, g_1^2, g_1^4, g_2^1, g_2^2, g_2^4\}$. The i and j in g_i^j represents the types of scattering processes in the axis direction and the circumferential direction; 1=interbranch backward, 2=interbranch forward, 3=Umklapp, 4=intrabranch forward. From the momentum transfer by phonons, we can set bare values of the couplings to be $g_1^2 = g_1^4 \equiv b$, $g_2^2 = g_2^4 \equiv f$.

When we neglect scatterings which change the circumferential momentum ($g_1^1 = g_2^1 = 0$), we can solve scaling equations exactly. Phase diagram for this case is shown in Fig. 1(a). In the gapful phase, CDW with zero circumferential momentum dominates other orders. For the gapless phase, TS and SS appears in the $b - 4f > 0$ domain, CDW and SDW appears in the $b - 4f < 0$ domain.

In order to clarify the role of scattering channels with circumferential momentum transfer (g_1^1, g_2^1), we show phase diagrams in (g_1^1, g_2^1) space [Fig. 1(b)] with constant b and f. The gapless phase is sensitive to g_1^1, g_2^1 and tends to be gapful phase. The phase diagrams mainly consist of four gapful phases CDW01, CDW02, SS01 and SS02, regardless of (b, f) values. All of them has zero circumferential momentum, which is indicated by "0" after abbreviations. In addition, the last number "1" and "2" corresponds to symmetrical or antisymmetrical order with respect to inversion of the tube axis. The effect of f on the phase diagram is negligible. We can see domain of SS02 and CDW02 becomes larger as b gets larger.

Group 3 : $(\gamma, \gamma') = (2, 3)$

These channels have six independent couplings $\{g_1^1, g_1^3, g_1^4, g_2^2, g_2^3, g_2^4\}$. We denote the tube radius by R and absolute value of circumferential momentum for band 2 and 3 by l_2 and l_3. If $l_2 + l_3 = \frac{5\pi\hbar}{2\pi R}$, the coupling constants for the Umklapp processes in the circumferential direction, g_1^3 and g_2^3, become finite.

When we neglect the Umklapp scatterings ($g_1^3 = g_2^3 = 0$), we can solve scaling equations exactly. The phase diagram is similar to that of Solyom, and gapful phase consists of CDW and SS with $l_2 \pm l_3$.

To investigate the effect of the Umklapp scatterings (g_1^3, g_2^3), we show the phase diagrams in (g_1^3, g_2^3) space in Fig. 2, where $\{g_1^1, g_1^4, g_2^2, g_2^4\}$ are set to constants. We can see destruction of gapless phase by increasing g_1^3 and g_2^3. The area of gapless region becomes larger as g_1^1 and g_1^4 get larger. The phase diagrams mainly consist of four gapful phases CDWL1, CDWL2, SSL1 and SSL2. Every orders has finite circumferential momentum $L = l_2 + l_3 = \frac{5\pi\hbar}{2\pi R}$, which is indicated by "L" in the abbreviations. "1" and "2" has the same meaning as in Group 2.

In summary, we have found CDW or SS phase with zero or $L = \frac{5\pi\hbar}{2\pi R}$ circumferential momentum in the gapful phase. We have also found instabilities of gapless phase due to interactions with circumferential momentum transfer. The couplings with the circumferential momentum transfer, such as g_1^1, g_2^1, g_1^3 and g_2^3 are crucial for determining the symmetry of various orders. The degrees of freedom in the circumferential direction of CN will show us new aspects of low-dimensional systems which are absent in purely 1-D systems.

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