Nonlinear response in the vortex state of unconventional superconductors

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Abstract

The origin of the finite density of states at low energy in the mixed state is discussed. The validity of the Doppler-shifted quasiparticle spectrum as an explanation of the $\sqrt{H}$-dependence (H is the magnetic field) of the specific heat coefficient is investigated. This shift is shown to be absent owing to the nonlocal effect. Our result indicates that the suppression of the amplitude of the superconducting gap should be included to discuss the low energy property.

Key words: mixed state; density of states; the specific heat

1. Introduction

The superconductor with nodes in its gap shows the power-law behavior in the temperature dependence, like the nuclear spin lattice relaxation rate and the penetration depth. This behavior reflects the power-law energy dependence of the density of states. About ten years ago Volovik predicted that this power-law behavior also bring about the characteristic property in its external field dependence like the $\sqrt{H}$-dependence of the specific heat coefficient. [1] The origin of $\sqrt{H}$-dependence is the nonanalytic response by the superfluid velocity ($\boldsymbol{v}_s(\boldsymbol{r})$) of the local density of states,

$$N(\epsilon = 0, \boldsymbol{r}) = N_{\rm F}\frac{\pi|\boldsymbol{v}_{\bf F}\cdot\boldsymbol{v}_s(\boldsymbol{r})|}{2\Delta_0}. \qquad (1)$$

($\boldsymbol{v}_{\bf F}$ is the quasiparticle velocity at the Fermi surface.)

The similar behavior in the Meissner state has been predicted by Yip and Sauls. [2] They predicted that the correction term in the supercurrent, expanded by the vector potential ($\boldsymbol{A}$), shows the nonanalytic behavior. This nonanalytic behavior originates from the divergence of the perturbation expansion by the external field. Recently it is shown that the perturbation expansion converges owing to the nonlocal effect in the Meissner states.[3] Therefore the corresponding properties in the mixed state and the validity of the Doppler-shifted quasiparticle spectrum should be reexamined. This is presented in this paper.

2. The perturbation approach

Firstly we show how the Doppler-shifted quasiparticle spectrum is derived and why this is invalid. If we write the Gor'kov equation and perform the singular gauge transformation, the following derivative operator appears, $[\boldsymbol{\nabla}_{\boldsymbol{x}} + \frac{\boldsymbol{\nabla}_{\boldsymbol{R}}}{2} - {\rm i}\boldsymbol{v}_s(\boldsymbol{R}+\boldsymbol{x}/2)]^2$. Here x and R are the relative coordinate and the center of mass coordinate, respectively. If we decouple x and R, i.e. $[\boldsymbol{\nabla}_{\boldsymbol{x}} - {\rm i}\boldsymbol{v}_s(\boldsymbol{R})]^2$, the Doppler-shifted quasiparticle spectrum is derived. Therefore eq.(1) is derived by neglecting the nonlocal effect. It is also seen that the nonanalytic function (in the sense of the absolute value) originates from the divergence of the perturbation expansion owing to the absence of the nonlocal effect as follows.

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$$N(\epsilon=0,R) = -\frac{1}{\pi}\sum_k \mathrm{Im}G_k(R)$$

$$= N_{\mathrm{F}}\frac{\pi}{2\Delta_0}\int \mathrm{d}E\left[\delta(E)+\frac{1}{2}\frac{\mathrm{d}^2\delta(E)}{\mathrm{d}E^2}[\boldsymbol{v}_{\mathbf{F}}\cdot\boldsymbol{v}_s(\boldsymbol{R})]^2+...\right] \quad (2)$$

Next we consider the nonlocal effect. The uniform component of the superfluid velocity is zero, $\boldsymbol{v}_s(\boldsymbol{q}=\mathbf{0})=0$, because this is an odd function. Then we need to calculate the finite q-behavior. The Dyson equation is written as,

$$\hat{G}_k(q) = \hat{G^0_k}V\delta q,0 - \hat{G^0_{k+q/2}}\frac{1}{V}\sum_{q'}\hat{O}_{k+\frac{q-q'}{2}}(q')\hat{G}_{k+q'/2}(q-q') \quad (3)$$

Here $\hat{G^0_k}$ is the uniform Green function, V is the volume of the system and $\hat{O}_k(q) := \boldsymbol{v}_{\mathbf{F}}\cdot\boldsymbol{v}_s(\boldsymbol{q})\hat{\tau}_0$ ($\hat{\tau}_0$ is the 2×2 unit matrix). We consider only $\hat{O}_k(q)$ as the perturbation because our purpose here is to consider the validity of the Doppler-shifted quasiparticle spectrum. To compare with the experiments it is needed to include the effect which comes from the suppression of the amplitude of the superconducting gap.

The total DOS is written as $-\frac{1}{\pi}\mathrm{Im}\frac{1}{V^2}\sum_k \mathrm{Tr}\hat{G}_k(q=0)$. By expanding the $\hat{G}_k(q=0)$ up to the second order of $\hat{O}_k(q)$, the correction to the DOS is written as follows.

$$-\frac{1}{\pi}\mathrm{Im}\frac{1}{V^2}\sum_k \mathrm{Tr}\hat{G}^{(2)}_k(q=0) = \frac{N_{\mathrm{F}}}{2\Delta_0}\frac{1}{V^2}\sum_{q'\neq 0}|\boldsymbol{v}_{\mathbf{F}}\cdot\boldsymbol{v}_s(\boldsymbol{q'})|^2 \times \frac{2|\epsilon|}{\sqrt{X_{q'}(4\epsilon^2-X_{q'})}}[F_\epsilon(a,b)+F_\epsilon(a,-b) + F_\epsilon(-a,b)+F_\epsilon(-a,-b)] \quad (4)$$

Here $X_q = a^2+b^2$, $a=\boldsymbol{v}_{\mathbf{F}}\cdot\boldsymbol{q}+...$, $b=\boldsymbol{v}_{\boldsymbol{\Delta}}\cdot\boldsymbol{q}...$ ('...' means the higher order terms of q, $v_\Delta := \partial\Delta_k/\partial k$, $F_\epsilon(a,b) = \tanh^{-1}\frac{X_q+2\epsilon(a+b)}{\sqrt{X_q(4\epsilon^2-X_q)}}$ It can be seen that for small ϵ the correction term is proportional to $|\epsilon|$, and this is same as in the uniform case. Therefore the Doppler-shifted quasiparticle spectrum is invalid at low energy. $\boldsymbol{q'}$ in eq.(4) is a discrete variable, $(q'_x, q'_y) = 2\pi(n,m)/d$ (n,m is the integer and d is the vortex lattice spacing). Therefore the $|\epsilon|$-linear term exists below the crossover value, $\epsilon_c = \sqrt{((v_k\cdot q')^2+(v_\Delta\cdot q')^2)/2}$ with the smallest value of q', $|\boldsymbol{q'}|=2\pi/d$. ϵ_c is estimated as $\epsilon_c \simeq 2\pi^2\Delta_0\sqrt{H/H_{c2}}$. We estimate ϵ_c to compare with the experiment, which indicate the $\sqrt{H}$-like behavior of the specific heat coefficient. $\epsilon_c \simeq 900$K for $\Delta_0 = 200$K and $H_{c2} = 150T$ with $H = 10T$ around which the experiment is done.[4] This value of ϵ_c is much higher than that of the experiment, around $2-4$K. This indicates that the observed behavior of the specific heat coefficient does not originate from the Doppler-shifted quasiparticle spectrum.

3. Discussion

Our approach is so simplified that the qualitative comparison cannot be done. The following point is, however, firstly clarified. The $\sqrt{H}$-like behavior of the specific heat coefficient is considered as the sign of the unconventional superconductor after the prediction by Volovik. However it is known that the downward curvature of the specific heat coefficient has been observed also in the s-wave superconductors. Therefore it is doubted by some author that the downward curvature is the sign of the existence of nodes.[5] Then the question whether the finite DOS originates from the suppression of the amplitude or the gauge field like the superfluid velocity, arises. The numerical calculation of the Bogoliubov-de Gennes equation have not clarified this point because both effects are calculated in this scheme, and concluded that the same effect as that of Volovik is obtained.[6] In this paper we conclude that the Doppler-shifted quasiparticle spectrum does not exist owing to the nonlocal effect and the gauge field cannot induce the finite DOS at the low energy. Therefore the variation of the amplitude should be included to treat the low energy property of the mixed state.

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