Pseudogap Phenomena in the BCS Pairing Model

Satoshi Fujimoto [a,1]

[a] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

Abstract

We investigate pseudogap phenomena realized in the BCS pairing model with a long but finite interaction range. We obtain all-order self-energy corrections exactly. It is found that vertex corrections to the self-energy are crucially important for the pseudogap state.

Key words: pseudogap; BCS pairing model; superconductivity

1. Introduction

Recently, it has been proposed by some authors that the pseudogap phenomena observed in high T_c cuprates may be attributed to the superconducting fluctuation.[1] In the previous studies, the single particle self-energy is calculated by one-loop approximation (t-matrix approximation, diagrammatically expressed as Fig.1(a)) or self-consistent t-matrix approximation (expressed as Fig.1(b)). These calculations give some different results. For example, in the self-consistent one-loop approximation, the pseudogap of the DOS is destroyed near the Fermi level, and the quasiparticle peak is restored. On the other hand, such a restoration is not seen in the lowest-order one-loop approximation. Thus, it is desirable to examine the effects of vertex corrections, which are expressed as Fig.1(c). For this purpose, we investigate a simple model for which we can calculate all-order self-energy corrections in the temperature range where the superconducting fluctuation is Gaussian-like.

2. Model and method

The model Hamiltonian is given by

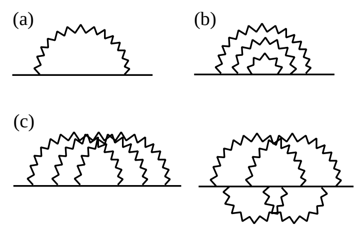

Fig. 1. Diagrams for the irreducible single-particle self-energy. The solid line represents the single electron Green's function. The wavy line represents the propagator of the superconducting fluctuation. (a) Lowest order one loop correction. (b) Self-consistent one loop approximation. (c) Some typical diagrams of vertex corrections which are not included in the diagrams (a) and (b).

$$H = \sum_{k,\sigma} E_k c^\dagger_{k\sigma} c_{k\sigma} - \frac{1}{N}\sum_q V(q) B^\dagger(q) B(q), \qquad (1)$$

where $V(q) = V\prod_{i=1}^d l_c/[\pi(1+l_c^2 q_i^2)]$, $B(q) = \sum_k \xi_k c_{k\downarrow} c_{-k+q\uparrow}$, and d is the spatial dimension. The second term is the pairing attractive interaction with the interaction range l_c. $B(0)$ is a local annihilation operator of a Cooper pair with a structure factor ξ_k. In the limit of $l_c \to +\infty$, eq.(1) is the pairing model, which is exactly solvable.[2,3] In this limit, the BCS mean field solution is exact, and the effects of fluctuation are completely suppressed. For finite but sufficiently large l_c, the superconducting fluctuation is restored, and can be calculated systematically using the expansion in terms of $1/l_c^d$. Note that $Vl_c^d \equiv U$

[1] Corresponding author. E-mail: fuji@scphys.kyoto-u.ac.jp

must be finite in the limit of $l_c \to \infty$ to keep the mean field T_c finite. In the limit of $l_c \to +\infty$, the superconducting fluctuation propagator is Gaussian-like,[3] $\langle \phi_{q,n}\phi^*_{q,n}\rangle = 1/(a|\omega_n| + bq^2 + t)$. Here $t = (T - T_c)/T_c$. In the case of $t \gg bl_c^{-2}$ for which we can calculate the all-order self-energy corrections, the Ginzburg criterion, $\sqrt{4T}/l_c^d \ll t$, is satisfied for $d \geq 2$, and thus, the superconducting fluctuation propagator is given by the Gaussian propagator with renormalized parameters even for finite but sufficiently large l_c. Using this propagator, we calculate the self-energy corrections.

3. All-order self-energy corrections

We have carried out the frequency sum in the calculation of the self-energy for all order diagrams. Then, we found that for $t \gg bl_c^{-2}$, the momentum sum is simplified, and that the self-energy has the same form as that obtained by Sadovskii for 1D fermion systems interacting with static Gaussian fluctuation.[4] Then, we can apply Elyutin-Sadovskii's combinatorics method. The self-energy in all orders is written as a continued fraction form,[5]

$$\Sigma_k(\varepsilon) = \frac{v(1)\tilde{g}_k \,\big|}{\big|\, S_1(k,\varepsilon)} - \frac{v(2)\tilde{g}_k \,\big|}{\big|\, S_2(k,\varepsilon)} - \frac{v(3)\tilde{g}_k \,\big|}{\big|\, S_3(k,\varepsilon)} - \cdots, \qquad (2)$$

where $\tilde{g}_k = T\xi_k^2 U/(t\pi^d l_c^d)$, and $S_m(k,\varepsilon) = \mathrm{i}\varepsilon + (-1)^{m-1}E_k + \mathrm{i}m(t/a)\mathrm{sgn}\varepsilon$, and $v(m) = [(m+1)/2]$, [...] is Gauss's symbol. Using this formula, we calculate the single-particle DOS, $\rho(\varepsilon)$, numerically in the case of s-wave pairing $\xi_k = 1$ (Fig.2(a)). The parameters are chosen so as to satisfy the condition $bl_c^{-2} \ll t$.

The pseudogap behavior is due to the large enhancement of the single-particle damping as shown in Fig.2(b), which is caused by strong scattering with superconducting fluctuation. To see how this enhancement is affected by the vertex corrections, we calculate analytically the damping at the Fermi level, up to a numerical constant, $\gamma \sim aTU/(\pi^d l_c^d t^2)$. For $\sqrt{aU/(\pi^d l_c^d)} \gg t$, the damping is much enhanced like $\sim T/t^2 \gg T$. Since we consider the case of $t \gg b/l_c^2$, the temperature range where the above result is applicable is $b/l_c^2 \ll t \ll \sqrt{aU/(\pi^d l_c^d)}$. As d increases, the tempearture range becomes narrower, and eventually vanishes for $d = 4$. It is noted that the result of all-order calculation gives the same asymptotic behavior as that obtained by the lowest-order one-loop approximation. This implies that higher order corrections cancel each other, and the lowest-order term gives the leading contribution. We compare this result with that obtained from self-consistent t-matrix approximation. The self-energy in the self-consistent approximation is derived by substituting $v(m) = 1$ for all m of eq.(2).

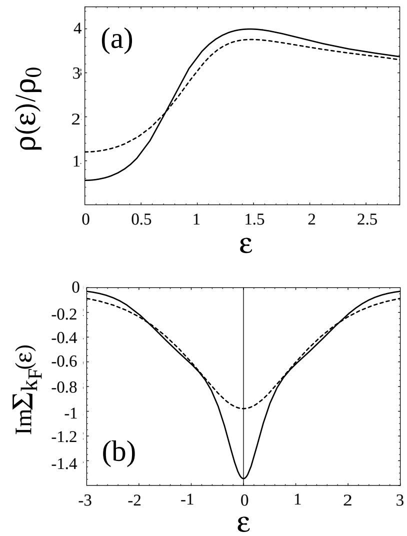

Fig. 2. (a) The single particle DOS $\rho(\varepsilon)$ plotted as a function of energy. The vertical axis is renormalized by the bare DOS ρ_0. $\tilde{g} = 1$, $C(0) = 0.1$ for the solid line. $\tilde{g} = 1$, $C(0) = 0.5$ for the dotted line. (b) The single-particle damping at the Fermi momentum plotted as a function of energy. The same parameters as above are used.

Then, we have $\gamma_{sc} \sim \sqrt{\tilde{g}_k}$ for sufficiently small t. For $t \gg b/l_c^2$, $\sqrt{\tilde{g}_k} \ll [\xi_k^2 UT/(\pi^d bl_c^{d-2})]^{1/2}$. Thus, in the case of $d = 2$ or 3, the damping γ_{sc} cannot be enhanced sufficiently. The deviation from the exact result is serious in this approximation These results imply that the lowest order one-loop calculation is more reliable than the self-consistent one-loop treatment.

In summary, It has been revealed that the vertex corrections to the self-energy are essentially important in higher order calculations of the pseudogap state.

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