Renormalization group approach to the energy level statistics at the integer quantum Hall transition

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Abstract

We apply the real-space renormalization group (RG) approach to investigate the energy level statistics at the integer quantum Hall (QH) transition. Within the RG approach the macroscopic array of saddle points of the Chalker-Coddington network is replaced by a fragment consisting of only five saddle points. Previously we have demonstrated that the RG approach reproduces the distribution of the *conductance* at the transition, $P(G)$, with very high accuracy. To assess the level statistics we analyze the *phases* of the transmission coefficients of the saddle points. We find that, at the transition, the nearest neighbor energy level spacing distribution (LSD) exhibits well-pronounced level repulsion. We emphasize that a metal-like LSD emerges when the *fixed point* distribution P_c of G is used. Studying the change of the LSD around the QH transition we observe scaling behavior. Using a one-parameter finite-size scaling analysis we are able to extract a critical exponent $\nu = 2.38 \pm 0.04$ of the localization length.

Key words: Integer quantum Hall effect; Metal-insulator transition; Renormalization group approach; Network model

1. Introduction

At the QH delocalization-localization transition, the localization length ξ of the electron wavefunction diverges like $\epsilon^{-\nu}$, where ϵ defines the energy distance to the transition and ν a critical exponent. The wavefunctions of the charge carriers change from being localized in the insulating to being extended in the metallic regime. The energy level spacing distribution (LSD) $P(s)$ describes the probability to find neighboring energy levels at an energy distance s. Since the localized wavefunctions are uncorrelated in space the corresponding LSD coincides with the Poisson distribution. On the metallic side spatial correlation leads to level repulsion for small s. The shape of $P(s)$ in the metallic regime is predicted by random matrix theory [1] and depends on the universality class the system belongs to. For our study, due to the broken time-reversal symmetry in a magnetic field, it is the Gaussian Unitary Ensemble (GUE). Exactly at the QH transition a third so called critical LSD $P_c(s)$ is found [2]. The discussion of the shape of $P_c(s)$ concentrates on the behavior in the tails. For small s it is commonly excepted that $P_c(s)$ resembles the level repulsion found in the corresponding metallic regime. For $s \gg 1$ two contradicting behaviors have been predicted: (i) $P_c(s) \propto \exp(-bs)$ [3] and (ii) $P_c(s) \propto \exp(-as^\gamma)$ [4], with $\gamma = 1 + (\nu d)^{-1}$ and the spatial dimension d. Besides the shape of $P_c(s)$, a finite-size scaling (FSS) approach using the LSD around the transition furthermore allows to evaluate ν [5], which is predicted to be a universal quantity.

In order to access the LSD at the QH transition we use a real-space RG approach to the Chalker-Coddington (CC) model [6] which has proven successful particular in the studies of the critical conductance distribution, the value of ν and the influence of long-range-correlated disorder [7].

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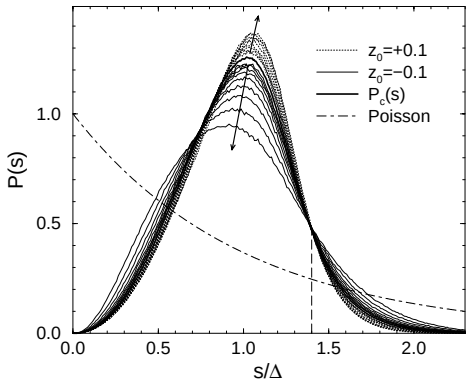

Fig. 1. Results for the RG of the LSD. $P(s)$ is rescaled by the mean level spacing Δ. The thick line shows $P_{\rm c}(s)$. The other curves corresponds to the first 9 RG iterations with an initial distribution P_0 shifted to the metallic ($z_0 = +0.1$) or the localized ($z_0 = -0.1$) regime. Within the RG procedure the LSD moves away from $P_{\rm c}(s)$ as indicated by the arrows. At $s/\Delta \approx 1.4$ the curves cross at the same point.

2. RG of the LSD

The CC model [6] describes a single QH transition using a chiral network consisting of electron trajectories along equipotential lines (*links*) and saddle points (SP's) of the potential (*nodes*). Each SP relates the wavefunction amplitudes in two incoming and two outgoing channels. The links correspond to a random phase Φ. From 5 such SP's we construct an RG unit, which describes the essential properties of the entire network [7,8]. Following from the size of the RG unit the effective system size of our sample is doubled in each RG step, which allows to reach very large systems. Starting from a initial distribution $P_0(t)$ of the transmission coefficient t of a SP we can now compute $P_1(t)$ of the RG unit and then continue iteratively [7]. We note that $P(G)$ of the dimensionless conductance G is related to $P(t)$ by $G = t^2$. The LSD $P_n(s)$ of RG step n is derived from a unitary network operator of the RG unit [9] which depends on $P_{n-1}(t)$ and $P_{n-1}(s)$. In order to obtain the eigenenergies we assume a linear energy dependence of the Φ's.

Using an initial $P_0(t)$ corresponding to the critical distribution $P_{\rm c}(G)$, we find [10] that $P_n(s)$ converges to the fixed point distribution $P_{\rm c}(s)$ within a few RG iterations. The shape of the bulk $P_{\rm c}(s)$, as shown in Fig. 1, does not agree very well with previous numerical studies. Nevertheless we can verify that the behavior for the small s tail resembles the predicted s^2 dependence. For $s \gg 1$ reasonable agreement of our data with the proposed shapes could be obtained for $P_{\rm c}(s) \propto \exp(-as^\gamma)$ with $\gamma = 2.4$.

The behavior of the LSD around the QH transition can be studied in terms of the scaling approach. For a second order phase transition $\xi_\infty(z_0) \propto |z_0 - z_{\rm c}|^{-\nu}$ for z_0 approaching the critical point $z_{\rm c}$. The parameter z_0 is a shift of SP energies inducing an initial shift of $P_{\rm c}(s)$ toward the localized ($z_0 < 0$) or the extended ($z_0 > 0$) regime, respectively. Two results of the RG iterations for $z_0 = -0.1, +0.1$ are shown in Fig. 1. The shape of the LSD allows to derive scaling quantities. In this work we choose $\alpha_{\rm P} = \int_0^{s_0} P(s)ds$ and $\alpha_{\rm I} = \frac{1}{s_0}\int_0^{s_0} I(s)ds$ [5], where $I(s) = \int_0^s P(s')ds'$. The integration limit $s_0 = 1.4$ marks a crossing point of all LSD curves, as can be seen in Fig. 1. We then compute ν by an FSS approach to $\alpha_{\rm P}$ and $\alpha_{\rm I}$ using a higher order nonlinear fit [11]. As result we obtain $\nu = 2.38 \pm 0.04$ in excellent agreement with previous numerical works [7,12].

3. Conclusion

Instead of a large CC network we use a small 5 SP RG unit to extract the eigenenergies. This approach uses a specific assumption of the energy dependence of the phases in the RG unit, which influences also the shape of the LSD. Therefore the overall form of the computed $P_{\rm c}(s)$ is not universal. But it still shows the predicted quadratic level repulsion for small s. The presence of universality is moreover verified by an FSS analysis of the LSD around the QH transition, where the exponent $\nu = 2.38 \pm 0.04$ of the localization length is obtained.

We argue that our findings indicate a large robustness of universal properties at the QH transition. Using a simple noninteracting semiclassical picture of electron propagation and the approximation of the RG approach we were able to reveal universal behavior.

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