# A theory of metallic conductivity of the two dimensional electron gas.

Boris Narozhny [a,1], Gabor Zala [b], Igor Aleiner [b]

[a] Physics Department, Brookhaven National Laboratory, Upton, NY, 11973, U.S.A

[b] Department of Physics, SUNY at Stony Brook, Stony Brook, NY, 11794, U.S.A

**Abstract**

It is well known that electron-electron interaction in two dimensional disordered systems leads to logarithmically divergent Altshuler-Aronov corrections to conductivity at low temperatures ( $T\tau \ll 1$; $\tau$ is the elastic mean-free time). This work is devoted to the fate of such corrections at intermediate temperatures $T\tau > 1$. We show that in this (ballistic) regime the temperature dependence of conductivity is still governed by the same physical processes as the Altshuler-Aronov corrections - electron scattering by Friedel oscillations. However, in this regime the correction is linear in temperature; the value and even the sign of the slope depends on the strength of electron-electron interaction (this sign change may be relevant for the “metal-insulator” transition observed recently).

*Key words:* metal-insulator transition; electron-electron interaction; Friedel oscillation;

Recent observations [1] interpreted as a metal-insulator transition in 2D electron systems challenged theoretical understanding of transport phenomena in disordered systems at low temperatures. Although the theory of quantum corrections due to Altshuler and Aronov [2] and further developed by Finkelstein [3] allows for the such sign change in the temperature dependence of the conductivity in 2D due to electron-electron interaction in the triplet channel, the prediction for the Hall coefficient [4] disagrees strongly with experiment. At the same time, the theory of temperature-dependent screening, first suggested by Stern [5] to describe data at higher temperatures, predicts the universal, metallic sign of the temperature dependence of conductivity. We address these contradictions by developing a unified picture of quantum interference effects in 2D electron gas, valid for all temperatures smaller than the Fermi energy. We show that the interaction corrections to conductivity arise due to coherent scattering of electrons off Friedel oscillations.

Consider an idealised problem of 2D electrons interacting via Coulomb interaction in the presence of point-like impurities, when the dimensionless conductance of the system is large $g \gg 1$. Although such a model does not take into account material-dependent details, such as valley degeneracy (relevant for experiments in *Si*-MOSFETs) or spin-orbit coupling (which is important in *SiGe* samples), it captures the essential interference phenomena that lead to temperature and magnetic field dependence of the transport coefficients.

In an interacting electron system one finds an interaction driven (i.e. absent in a free gas) interference effect, namely coherent electron scattering off Friedel oscillations. In the ballistic regime ($T\tau \gg 1$) one needs to consider only a single impurity. The impurity potential induces a modulation of electron density close to the impurity. The oscillating part of the modulation is known as the Friedel oscillation. The leading correction to conductivity is a result of interference between two semi-classical paths: (i) an electron scatters off the Friedel oscillation created by the impurity; (ii) it scatters off the impurity itself. Interference is most important for scattering angles close to $\pi$ (or for backscattering), since the extra phase factor accumulated by the electron on path (i) is canceled by the phase of the Friedel oscillation so that the amplitudes corresponding to the two paths are coherent. As a result, the probability of backscattering is greater than the classical

[1] Corresponding author. E-mail: narozhny@bnl.gov

expectation (taken into account in the Drude conductivity). At lower temperatures, one needs to consider multiple impurity scattering. In that case the Friedel oscillation is created by all impurities involved in the process. It is clear that such effect contributes to the scattering amplitude at any angle, which is typical of the diffusive motion of electrons. At the lowest temperatures scattering off Friedel oscillations leads to the logarithmic temperature dependence of the conductivity [2]. We note that interference persists to large distances, limited only by temperature $R \approx 1/|k - k_F| \leq v_F/T$. Thus there is a possibility for the resulting conductivity correction to have a non-trivial temperature dependence. The sign of the correction depends on the sign of the coupling constant that describes electron-electron interaction. For the above simplest model that is the Fermi liquid constant $F_0^\sigma$, which describes the "triplet" channel interaction.

The above physical picture yields the following results (details of calculations can be found in Ref. [6]). In the absence of the magnetic field the total correction to the conductivity can be written as a sum of the "charge" and triplet contributions

$$\sigma = \sigma_D + \delta\sigma_T + \delta\sigma_C, \tag{1}$$

where the charge channel correction is given by

$$\delta\sigma_C = \frac{e^2}{\pi\hbar}\frac{T\tau}{\hbar}\left[1 - \frac{3}{8}f(T\tau)\right] - \frac{e^2}{2\pi^2\hbar}\ln\frac{E_F}{T}, \tag{2}$$

and the triplet channel correction is

$$\delta\sigma_T = \frac{3F_0^\sigma}{(1+F_0^\sigma)}\frac{e^2}{\pi\hbar}\frac{T\tau}{\hbar}\left[1 - \frac{3}{8}t(T\tau; F_0^\sigma)\right] - 3\left(1 - \frac{1}{F_0^\sigma}\ln(1+F_0^\sigma)\right)\frac{e^2}{2\pi^2\hbar}\ln\frac{E_F}{T}. \tag{3}$$

Here the factor of 3 in the triplet channel correction Eq. (3) is due to the fact that all three components of the triplet state contribute equally. The dimensionless functions $f(x)$ and $t(x)$ describe the cross-over between ballistic and diffusive regimes and can be disregarded for the qualitative discussion.

The temperature dependence of the Hall coefficient can be approximated by the following expression

$$\frac{\delta\rho_{xy}}{\rho_H^D} = \frac{e^2}{\pi^2\hbar\sigma_D}\ln\left(1 + \frac{11\pi}{192}\frac{\hbar}{T\tau}\right) \times \left[4 - \frac{3}{F_0^\sigma}\ln\frac{1 + F_0^\sigma + g_1(F_0^\sigma)T\tau/\hbar}{1 + T\tau/\hbar}\right], \tag{4}$$

where $\ln g(x) = (4/11)[-5f_3(x) - 12f_2(x) - 3f_1(x) + 4f_0(x)]$, with $f_j(x) = (\ln[1+x] + \sum_{n=1}^{j}(-x)^n/n)/x^j$. In the ballistic limit the correction decays with temperature $\delta\rho_{xy} \propto 1/T\tau$ which might explain the unusual results of recent experiments [1].

In parallel magnetic field electrons acquire additional Zeeman energy $E_z = g\mu_B H$, which is proportional to the magnitude $H$ of the field, the Bohr magneton $\mu_B$, and the electron $g$-factor. Consider for simplicity the ballistic regime. If $E_z > T > \tau^{-1}$ then the two components of the triplet channel are frozen by the magnetic field. This results in the change in the slope of the temperature dependence as well as in the magneto-conductivity

$$\delta\sigma(H,T) = \frac{e^2}{\pi\hbar}\left[\frac{2 + F_0^\sigma}{1 + F_0^\sigma}\frac{T\tau}{\hbar} + \frac{F_0^\sigma g(F_0^\sigma)}{1 + F_0^\sigma}\frac{E_z\tau}{\hbar}\right] \tag{5}$$

where $g(z) = \frac{1}{2z}\ln(1+z) + \frac{1}{2(1+2z)} + \frac{z\ln 2(1+z)}{(1+2z)^2}$. At the strongest fields $E_z^* > E_F$ when the system is fully polarized the spin does not play a role any more and one retrieves the universal singlet channel result

$$\partial\sigma/\partial T = e^2\tau/\pi\hbar^2; \qquad T\tau/\hbar > 0.1; \quad E_z^* > E_F. \tag{6}$$

This result is in agreement with recently reported measurements in $GaAs$ heterostructures [7].

In conclusion, for interacting disordered 2D electron systems we find temperature and magnetic field dependence of transport coefficients. All independently observable quantities are obtained in terms of the same set of parameters, allowing us to predict results of future measurements and gain insight into the microscopic structure of the interacting electron system.


## Acknowledgements

One of us (I.A.) is supported by the Packard foundation. Work at BNL is supported by the US DOE under contract number DE-AC02-98 CH 10886.


## References


[1] E. Abrahams, S.V. Kravchenko, M.P. Sarachik, Rev. Mod. Phys. **73** (2001) 251.

[2] B.L. Altshuler, A.G. Aronov in *Electron-Electron Interactions in Disordered Systems*, eds. A.L. Efros, M. Pollak (North-Holland, Amsterdam, 1985).

[3] A.M. Finkelstein, Zh. Eksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys. JETP **57**, 97 (1983)]; Z. Phys. B **56**, 189 (1984).

[4] B.L. Altshuler , D.E. Khmelnitskii, A.I. Larkin, P.A. Lee, Phys. Rev. B **22**, 5142 (1980).

[5] F. Stern, Phys. Rev. Lett. **44**, 1469 (1980); F. Stern, S. Das Sarma, Solid State Electron., **28**, 158 (1985); A. Gold and V.T. Dolgopolov, Phys. Rev. B **33**, 1076 (1986); S. Das Sarma, E.H. Hwang, Phys. Rev. Lett. **83**, 164 (1999).

[6] Gabor Zala, B.N. Narozhny, I.L. Aleiner, Phys. Rev. B **64**, 214204 (2001); *ibid* **64**, 201201 (2001); *ibid* **65**, 020201 (2001).

[7] Y.Y. Proskuryakov *et.al*, cond-mat/0109261.