Two-particle pairing in 2D Bose gases.

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Abstract

We consider a possibility of two-boson pairing in a dilute 2D Bose-gas with strong hard-core repulsion and a Van der Waals attraction tail. We show that the phase diagram for a dilute Bose-gas of one sort structureless bosons consists of two regions. The first one is the region of usual one-particle Bose-Einstein condensation. But the second is the region of total phase separation on Mott-Hubbard Bose-solid and dilute Bose gas. However already for Bose-gas consisting of two sorts of structurless bosons described by the two band Hubbard model the s-wave pairing of the two bosons of different sorts $\langle b_1 b_2\rangle \neq 0$ is possible.

Key words: 2D; BEC; cooper pairing

1. Introduction.

The problem of Bose-Einstein condensation (BEC) currently has a great interest. One of the intrigue question concerning Bose-Einstein condensation is possibility of its realization in the two dimensional case. The nature of BEC in 2D is different from the three-dimensional case. As the temperature of BEC for ideal Bose gas in 2D is zero, it can take place only due to interactions between particles or an external field. Besides this in 2D Bose gas in contrast to 3D it is possible to realize two-particle pairing [1-3]. It means that $\langle b\rangle = 0$, while $\langle bb\rangle \neq 0$. The aim of the present paper is a discussion of realization of two-particle condensation for continuous case and possibility of experimental realization in magnetic traps.

2. Two-particle pairing.

As it was shown in [3,4] the phase diagram of 2D Bose gas of structurless particles on a square lattice with nearest neighbors attraction and hard-core repulsion has two region. One of the region is usual BEC. The second region is the region of phase separation. So there is no region of two particle pairing in the lattice model. Now we want to analyze the possibility of realization of two particle pairing in a continuous model. We will consider a Bose gas with hard-core repulsion and a van der Waals attractive tail.

To consider the possibility of two particle condensate in continuous model we should find the parameters of appearance of a bound state in the two particle problem. To obtain it we need to solve Schrödinger equation for the potential with a strong hard-core and a weak Van der Waals tail $U \gg \{V; 1/mr_0^2\}$ (see Fig. 1).

The equation for the energy spectrum reads:

$$\frac{\kappa(Y_0(\kappa r_0)J_1(\kappa r_1)) - J_0(\kappa r_0)Y_1(\kappa r_1))}{(Y_0(\kappa r_0)J_0(\kappa r_1)) - J_0(\kappa r_0)Y_0(\kappa r_1))} = \frac{kK_1(kr_1)}{K_0(kr_1)}, \quad (1)$$

where J_0, Y_0 and J_1, Y_1 are the Bessel functions of zeroth and first order, K_0 and K_1 are the Macdonald functions of zeroth and first order. In (1) we introduce: $k = \sqrt{m|E_b|}$ and $\kappa = \sqrt{m(V - |E_b|)}$.

We are interested in the lowest threshold for the appearance of an s-wave pairing. Hence we can put $E_b = 0$ in the definition of κ. The first solution of (1) for $k = 0$ appears when $\kappa r_0 \geq 0.4$ and $\kappa r_1 \geq 1.6$.

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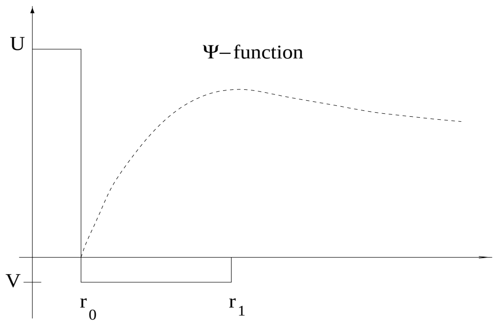

Fig. 1. The Ψ-function of an extended s-wave pairing. r_0 is a radius of a hard core repulsion, r_1 is a radius of a Van der Waals attraction. On the lattice $r_0 \sim d/2$ and $r_1 \sim d$

It means that an attractive tail must be four times more extended than a hard-core ($r_1 \approx 4r_0$). The Ψ-function in this case monotonously increase in the interval between r_0 and r_1, so we can say, that r_1 is the mean distance between the two bose particles. The threshold condition can be represented as $m|V_c|r_1^2 = 2.6$, where mr_1^2 corresponds to $1/2t$ in a lattice model. It means that a threshold in a continuous model is larger than in a lattice model. Hence the situation in continues model remains qualitatively the same as in the model of structurless Bose gas on a lattice. The phase separation at $T = 0$ is energetically preferable in comparison with two particle pairing.

The real possibility of obtaining two-particle pairing is connected with consideration of two sorts of bosons with hard core repulsion within bosons of the same sort and attraction between bosons of different sorts or two layers situation (see for details [4]). These situations are described by the Hamiltonian:

$$H = \sum \varepsilon_p a_p^\dagger a_p + \sum \varepsilon_p b_p^\dagger b_p + U_{aa} \sum a_{p_1}^\dagger a_{p_2}^\dagger a_{p_3} a_{p_4}$$
$$+U_{aa} \sum b_{p_1}^\dagger b_{p_2}^\dagger b_{p_3} b_{p_4} - U_{ab} \sum a_{p_1}^\dagger b_{p_2}^\dagger a_{p_3} b_{p_4},$$

where $\varepsilon_p = p^2/2m$. For simplicity we will consider the case of equal masses and densities. In the Hamiltonian U_{aa} and U_{bb} are hard-core repulsions for bosons of sorts a and b. And U_{ab} is attraction between bosons of two different sorts. In this case where is no phase separation and no thresholds for the attractive potential.

In the weak-coupling limit $U_{ab} \ll W$, where $W = 2/mr_1^2$ being the analog of the band in the lattice model, the bound state energy is given by the following expression:

$$|E_b| \sim W \exp\left\{-\frac{4\pi}{U_{ab}}\right\}.$$

The temperature of the two-particle pairing again has the form: $T_c \sim T_0/\ln(mU_{ab}/4\pi)$, where $T_0 = 2\pi/m$ is degeneracy temperature, n is density of the Bose gas. Note that $E_b < T_c < T_0$. For $0 < T < T_c$ the spectrum of quasiparticles $E_p = \sqrt{(\varepsilon_p + |\tilde{\mu}|)^2 - \Delta^2}$ acquires a gap $E_g = \sqrt{\tilde{\mu}^2 - \Delta^2}$, where $\tilde{\mu}$ is chemical potential and Δ is order parameter. At $T = 0$ we have $|\tilde{\mu}| = \Delta$ and the spectrum is soundlike.

In the strong coupling case $U_{ab} \gg W$ we have:

$$|E_b| \approx U_{ab} \gg T_0.$$

And the Bose gas first organizes local bosonic pairs. The corresponding crossover temperature is $T \sim |E_b|/\ln(W/T_0)$. And at lower temperature the pairs are Bose condensed.

The obtained result can be applied for anisotropic magnetic traps where one of the oscillator frequencies is large:

$$\{\Omega_x; \Omega_y\} \ll T_c < (T_0 \sim \Omega_z),$$

Then the system occupies only the lowest energy level in z-direction and becomes effectively two-dimensional. In this case formula for T_c are qualitatively correct, because the coherence length of the bose gas ξ_0 is smaller than an effective size of the trap $R(\varepsilon)$ for $\varepsilon \sim T_c$. Another limitation on the two-particle pairing is connected with the energy release of the order of $Q \sim |E_b|$ when the pairs are created. Due to this energy release the most energetical particles can overcome the potential barrier and evaporate from the trap (a process which is analogous to an evaporative cooling). However for $|E_b| < T_0$ the fraction of the escaping particles is small, so the superfluid state will be long living in this case.

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