Numerical studies of the superfluid Shapiro effect

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Abstract

Although early theoretical descriptions of the Shapiro effect focused on a voltage biased Josephson junction, the first experimental results were current biased. A superfluid ^{3}He system can be used to strictly pressure bias a Josephson weak link in an analogous way to voltage biasing a superconducting system. Here, we use numerical methods in an effort to reproduce key features found in the observed "superfluid Shapiro effect".

Key words: Superfluid;Weak Links;Josephson Effect;Shapiro Effect

In his original paper[1], Josephson proposed that if a constant voltage bias plus an ac voltage is applied across a superconducting Josephson junction, the super-currents will exhibit characteristic changes. Shapiro first observed these phenomena using current-biased superconducting Josephson junctions.[2,3] Nearly 40 years later, the "superfluid Shapiro effect" has been observed using a pressure biased superfluid ^{3}He weak link array.[4] Here, we focus on numerical methods to reproduce key features in this data.

In our double-diaphragm superfluid system, we provide a constant pressure bias using the "lower" diaphragm and a previously developed feedback technique.[5] We can provide an additional ac pressure excitation using the "upper" diaphragm. The resultant pressure seen across the weak link array should be

$$P(t) = P_{dc} + P_{ac}\cos(\omega t + \varphi) \tag{1}$$

Through the Josephson relations ($I = I_c \sin(\phi)$ and $\dot{\phi} = -2m_3 P/\rho\hbar$), we expect the pressure (1) will produce new currents of the form

$$I_n = I_c\,|J_n(\gamma)| \tag{2}$$

where $\gamma = 2m_3 P_{ac}/\rho\hbar\omega$ and J_n is the Besssel function of nth order. This leads to two types of Shapiro effects: (i) A *reduction* in the critical current of the superfluid weak link array ($n = 0$), (ii) an *increase* in the dc currents (spikes) at pressures which satisfy the condition

$$\frac{P_{dc}}{\rho} = n\frac{\hbar\omega}{2m_3} \tag{3}$$

for $n > 0$.

Measurements of the low amplitude pendulum mode oscillations when $I \approx I_c \sin(\phi)$ have confirmed the predictions of (i). To verify (ii), we observed additional dc currents in the I-P characteristic for different amplitudes of the ac excitation. Fig. 1 shows a sharp "feature" in the dc current centered about Josephson frequencies equal to the ac excitation frequency, $\omega_J/2\pi = \omega/2\pi = 105$ Hz in accordance with (3) where $n = 1$. These additional currents are seen (in panel **a**) to increase and then decrease as the amplitude of the ac excitation is increased (from the bottom curve to the top curve). Panel **b** shows that the size of this feature varies in accordance with (2). The shape of the features found in the I-P characteristics are not merely "current spikes" as predicted by the simple theoretical model (2) first derived by Shapiro.[2] This implies that the pressure across the weak link is *not* given by (1).

Consider a situation where the pressure from (1) is applied across both the superfluid weak link *and* a series ohmic resistance Z. The solution for the phase difference $\phi(t)$ must come from

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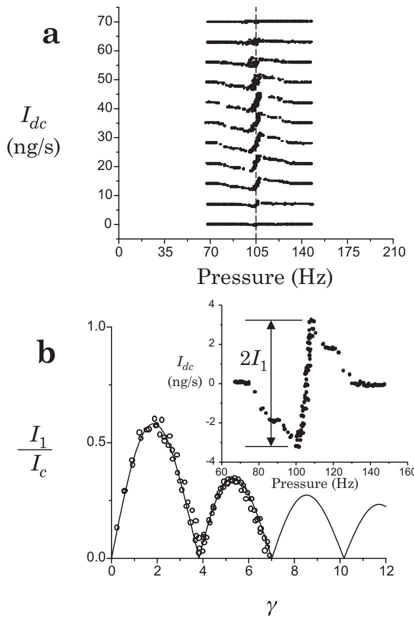

Fig. 1. (**a**) A plot of a series of I-P characteristics showing the current "feature". (**b**) A plot of I_1 as a function of γ.

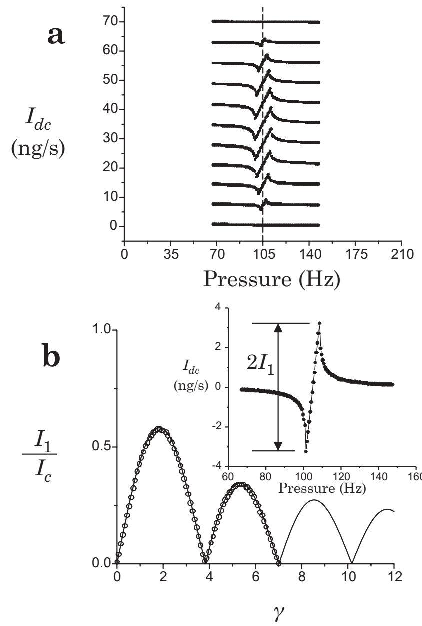

Fig. 2. (**a**) A plot of the prediction for the I-P characteristic. (**b**) A plot of I_1 as a function of γ.

$$\dot{\phi} = \omega_Z \sin(\phi) - \omega_J - \gamma\omega\cos(\omega t + \varphi) \tag{4}$$

where $\omega_Z = 2m_3 I_c Z/\hbar$. Eq. (4) can be solved numerically using a 4th order Runge-Kutta technique. Once $\phi(t)$ is known, the resulting additional dc currents are given by

$$I_n = I_c \langle \sin(\phi) \rangle \tag{5}$$

Numerical simulations show that as the magnitude of the resistance Z is increased, the current spike transforms into a tilted "S"-shape like that found in the data. Panel **a** in Fig. 2 shows the numerical results when $n = 1$, $\omega/2\pi = 105$ Hz and we have chosen $\omega_Z/2\pi = 11$ Hz so that the slope of the central slant in the feature is nearly equal to that of the experimental data. We find an impressive agreement between the data and the resulting shape of the prediction made using (4) and (5). The size of this feature increases and decreases with γ in a consistent way with the theoretical prediction (2) as shown in panel **b**.

The slope of the feature is a result of the difference between the dc pressure applied across the whole system and that found across the weak link. The time average of $\omega_Z \sin(\phi)$ in (4) shifts the dc pressure across the weak link, $\propto \langle \dot{\phi} \rangle$, from the applied value, $\propto \omega_J$. If we were to plot the dc current as a function of $\langle \dot{\phi} \rangle$ the feature would appear vertical because of the locking condition, $\langle \dot{\phi} \rangle = \omega$, during the increasing dc currents.

Although we have found very good agreement between these new theoretical results and the experimental data, it is unclear whether the proposed ohmic resistance Z *actually* exists in series with the superfluid weak link array. This is because its origin is unknown and the value needed for Z to produce the proper agreement with the data is three times larger than known sources of dissipation. However, it is clear that the effect of the series resistance Z is to alter the pressure found across the weak link from a simple sinusoid. For now, we take this result as an indication that the resulting ac pressure seen across the weak link is distorted from the sinusoid we have applied to the upper diaphragm in the experimental cell.

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