Physics of Vortex Core in Chiral P-wave Superconductor

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Abstract

We describe the essence of quantum effects inside s-wave and chiral p-wave vortices, without explicit use of quasiclassical Green function formalism. Physical quantities such as the impurity scattering rate and nuclear spin relaxation rate contain the coherence factor of the Andreev bound states in the matrix element of the transitions. The coherence factor of the Andreev bound state is different from that of a quasiparticle in bulk superconductors. Consequently, the physics within vortex core is different from that in normal state or that in spatially uniform superconducting state.

Key words: vortex core, chiral *p*-wave superconductor, coherence effect, impurity scattering

Vortex cores are often regarded as locally realized normal regions surrounded by a bulk superconducting state. This picture is valid in dirty superconductors where the mean free path l is much shorter than the coherence length ξ_0 at zero temperature. In clean superconductors where $l \gg \xi_0$ is satisfied, however, this "normal core " picture is no longer valid. Here we present the results on impurity scattering rates in 2D s-wave and chiral p-wave[1,2] vortices, in which quantum effects intrinsic to vortex cores become manifest[3,4]. We also discuss briefly the nuclear relaxation rate within vortex core[5,6]. While earlier studies[4] have been presented in the Green function formalism, the style of presentation in this paper adheres to that in [7] in spirit, in order to make the physics more accessible.

For simplicity, we consider superconductors with isotropic Fermi surface and axisymmetric vortex under the pair-potential with stepwise profile:

$$\Delta(\boldsymbol{r},\alpha) = \begin{cases} \Delta_0 \mathrm{e}^{\mathrm{i}N\alpha}\,(x+\mathrm{i}y)\,/r, & \text{for } \xi_0 < r \\ 0, & \text{for } 0 < r < \xi_0 \end{cases} \qquad (1)$$

where α is the angle with respect to x-axis in 2D momentum space and N denotes *chirality*[4]; N is zero for s-wave and ± 1 for chiral p-wave.

When the condition $k_{\mathrm{F}}\xi_0 \gg 1$ is satisfied, then the motions of quasiparticles are described by wave-packet. We first consider the pure case and then take into account the effect of impurities. In the core, a quasiparticle moves back and forth on a line (trajectory) with direction α (Fig. 1(a)) changing particle-hole channel on the Andreev reflection at the wall of pair-potential. Constructive interference of multiple Andreev reflections yields an Andreev bound state. This situation is similar to the S-N-S junction[8]. Using this similarity, the energy of the Andreev bound state on a trajectory with impact parameter b with respect to vortex center is given by $E(b) \sim \Delta_0 b/\xi_0$. This result is followed by the local density of states (DOS); $N(\varepsilon,\boldsymbol{r})/N_0 = \xi_0/\sqrt{r^2 - b_\varepsilon^2}$ for $r > b_\varepsilon \equiv \varepsilon\xi_0/\Delta_0$ and zero otherwise. Here N_0 is DOS in normal state. Now we introduce the coordinate s along a trajectory $\boldsymbol{r} = s(\cos\alpha,\sin\alpha) + b(-\sin\alpha,\cos\alpha)$. The wave function of the Andreev bound state is then given by

$$\begin{pmatrix} u_\alpha(s) \\ v_\alpha(s) \end{pmatrix} = \mathrm{e}^{\mathrm{i}k_{\mathrm{F}}s} \begin{pmatrix} \mathrm{e}^{\mathrm{i}(1+N)\alpha/2} \\ -\mathrm{i}\mathrm{e}^{-\mathrm{i}(1+N)\alpha/2} \end{pmatrix}.$$

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From these results, we can obtain the scattering rate due to randomly distributed impurities. The scattering rate $\Gamma(\varepsilon)$ of quasiparticle forming a bound state with energy ε and momentum direction α is given by[4]

$$\frac{\Gamma(\varepsilon)}{\Gamma_{\rm n}} \sim \int_{-\xi_0}^{\xi_0} \frac{{\rm d}s}{\xi_0} \frac{N(\varepsilon, \boldsymbol{r})}{N_0} \left| u_\alpha u^*_{\alpha'} - v_\alpha v^*_{\alpha'} \right|^2 \tag{2}$$

within Born approximation. $\Gamma_{\rm n}$ is the normal state scattering rate and α' is the momentum direction of the out-going state. In the integral, $\boldsymbol{r}$ runs along a trajectory of the in-going state with fixed value of b_ε. The coherence factor in (2)

$$\left| u_\alpha u^*_{\alpha'} - v_\alpha v^*_{\alpha'} \right|^2 /4 = \sin^2 \left[(1+N) \left(\alpha - \alpha' \right) /2 \right] \tag{3}$$

depends on the scattering angle $\alpha' - \alpha$ and N. From the geometrical relation shown in Fig. 1(b), this matrix element (3) is given by $s/\sqrt{s^2+b_\varepsilon^2}$ for $N=0$, $2sb_\varepsilon/(s^2+b_\varepsilon^2)$ for $N=1$ and zero for $N=-1$. Combining these results with $N(\varepsilon,\boldsymbol{r})/N_0 \sim \xi_0/|s|$, we obtain

$$\Gamma/\Gamma_{\rm n} \sim \begin{cases} \ln\left(\Delta_0/\varepsilon\right), & \text{for } N=0 \text{ (s-wave)} \\ \mathcal{O}(1), & \text{for } N=1 \text{ (chiral p-wave)} \\ 0, & \text{for } N=-1 \text{ (chiral p-wave).} \end{cases} \tag{4}$$

While the local DOS in the core are common in the three cases $N=0,\pm1$, the scattering rates (4) are qualitatively different. We can see, from this result, that the coherence effect plays an important role in the scattering process within vortex cores. The scattering rate for $N=-1$, in particular, vanishes owing to the destructive coherence effect, although there are available out-going states within vortex cores.

Similarly, the nuclear spin relaxation rate[5,6] is also subject to the coherence effect. For chiral p-wave case ($N=\pm1$), the nuclear spin relaxation rate at $\boldsymbol{r}$ *near and off* the vortex center is given by

$$1/T_1(\boldsymbol{r}) = \frac{\pi}{8} \int_0^\infty \frac{{\rm d}\varepsilon N^2(\varepsilon,\boldsymbol{r}) \left| u_\alpha u^*_{\alpha'} - v_\alpha v^*_{\alpha'} \right|^2}{\cosh^2(\varepsilon/2T)} \tag{5}$$

with the same coherence factor as that in (3) (We note that Eq. (5) is not valid for $N=0$). $1/T_1$ *vanishes* for $N=-1$, even in the presence of quasiparticles with finite local DOS.

Vanishing impurity scattering rate and nuclear spin relaxation rate for chiral p-wave vortex with $N=-1$ are the most conspicuous quantum effects in vortex cores. Obviously, both rates are finite in the bulk chiral p-wave superconductors. These results explicitly show that a vortex core has intrinsic quantum phenomena, which are absent in normal cores or bulk superconductors.

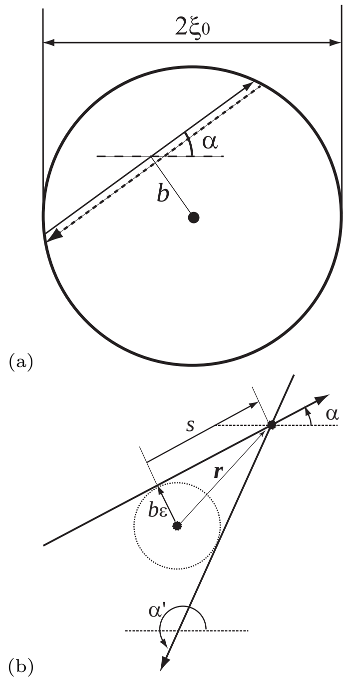

Fig. 1. (a) Schematics of the Andreev bound states formed by particle (represented by solid line with an arrow) and hole (dashed line) with momentum direction α in the vortex core with a stepwise radial profile (Eq. (1)). b denotes the impact parameter of the trajectory with respect to vortex center. (b) Geometrical relation in the scattering event at $\boldsymbol{r}$ from the in-going state α to the out-going state α'. s is the coordinate along the trajectory.

We thank M. Ichioka, M. Takigawa, M. Matsumoto, K. Machida, N. Schopohl and M. Sigrist for helpful discussions.

References

[1] M. Sigrist *et al.*, Physica C **317-318** (1999) 134.

[2] Y. Maeno *et al.*, Nature **372** (1994) 532.

[3] G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **70** (1999) 601 [JETP Lett. **70** (1999) 609].

[4] Y. Kato, J. Phys. Soc. Jpn. **69** (2000) 3378; Y. Kato and N. Hayashi, *ibid* **70** (2001) 3368; *ibid* **71** (2002) No. 7; N. Hayashi and Y. Kato, Physica C **367** (2002) 41; cond-mat/0202191.

[5] M. Takigawa *et al.* Phys. Rev. Lett. **83** (1999) 3057; J. Phys. Soc. Jpn. **69** (2000) 3943; to be published in J. Phys. Chem. Solids.

[6] N. Hayashi and Y. Kato, The Proceedings of LT23.

[7] M. Stone, Phys. Rev. B **54** (1996) 13222.

[8] I. O. Kulik: Zh. Eksp. Teor. Fiz. **57** (1969) 944 [Sov. Phys. JETP **30** (1970) 944].