Competing Orders and Field Induction of D+iD' State

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Abstract

The role of the magnetic field on the d-wave density wave as a model of pseudogap state of cuprates and on the d-wave superconducting state will be addresses. We argue that in d-wave density state magnetic field can produce secondary gap components. This distortion by magnetic field offers a possibility to distinguish between different scenarios of pseudogap in normal state of high-Tc materials. Similarly we argue that magnetic field can distort the p-wave state and produce secondary component of the gap in p-wave superconductor.

Key words: d+id' superconductor, d density wave, pseudogap, competing order, field induced component

1. Introduction

We will argue that the ground state of a many body system can be distorted and a secondary component of the order parameter can be generated upon applying magnetic field. This distortion, if it produces a new nontrivial component of the order parameter, leads to the symmetry lowering of the state.

We start with the general symmetry arguments on why magnetic field can lower the symmetry of the state. Consider a general many body ground state $|\Psi_0\rangle$. Now consider this state in the external magnetic field $B||z$, where we take field to be along z-axis. Here we will focus on the orbital effect of magnetic field. The relevant interaction term in the Hamiltonian of the system is:

$$H_{int} = \hat{M}_z B \tag{1}$$

where orbital magnetic moment operator $\hat{M}_z = g\mu_B \hat{L}_z$ with $g-$ being the gyromagnetic ratio and μ_B is the Bohr magneton. The first order perturbation theory gives for the correction to the ground state:

$$|\Psi_1\rangle = H_{int}|\Psi_0\rangle \tag{2}$$

There are three possibilities for $|\Psi_1\rangle$: i) $|\Psi_1\rangle$ is zero. In this case there is no linear effect of the field on the ground state. Symmetry of the state is not lowered. This is the case, as can be verified directly, when we apply magnetic field to s-wave superconductor. The angular momentum operator applied to ground state yields zero; ii) $|\Psi_1\rangle$ is collinear with $|\Psi_0\rangle$. The effect of the field to this order is only to change the amplitude of the state, e.g. the magnitude of the order parameter. Again symmetry is not lowered in this case; iii) case when $\langle\Psi_1|\Psi_0\rangle = 0$. In this case the newly acquired amplitude $|\Psi_1\rangle$ is of different symmetry, the ground state is *distorted* by magnetic field $|\Psi_0\rangle \rightarrow |\Psi_0\rangle + |\Psi_1\rangle$ and *symmetry of the ground state is lowered in the magnetic field*. It is not surprising that magnetic field can produce a new component of the order parameter. For example d-wave state is time reversal invariant. In the magnetic field, since time reversal is explicitly broken, the state becomes a $d_{x^2-y^2} + id_{xy}$ with finite angular momentum.

Below we will focus on the only nontrivial case iii). We will specifically consider the case of d-wave density wave as a model for a pseudogap state of cuprates. We also mention the case of unconventional p-wave superconductors.

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2. Pseudogap state and magnetic field induction of secondary component

The nature of the competing orders in high-Tc phase diagram and the pseudogap phase of underdoped cuprates is an important issue that is one of the most strongly debated questions. Some models attribute PG to superconducting phase fluctuations above T_c [1–3]; or to a competing non-superconducting order parameter [4,5].

We propose here to use magnetic field as a test of the symmetry of pseudogap state. If indeed the normal state of cuprates is a D wave density wave (DDW), as proposed in [5], it will have a nontrivial response to magnetic field. We argue that, in addition to the dominant $d_{x^2-y^2}$ (d) component of the DDW order parameter, a subdominant d_{xy} (d') component can be generated by the magnetic field. The fully gapped particle spectrum and the magnetically active collective mode of the condensate are the experimentally relevant consequences of $d+id'$ density wave state. Detailed discussion for DDW case is also given in [6]. Similar phenomenon of field induction of secondary component for d-wave superconductor was also considered, e.g. in [7]. The physical origin of this instability is the bulk orbital magnetic moment $\langle M_z\rangle$ in the $d+id'$ state. When an external magnetic field H is applied perpendicular to the plane of the two-dimensional (2D) system under consideration (namely, $\mathbf{H} \parallel \hat{\mathbf{z}}$), the resulting coupling of the magnetic induction B with the orbital magnetic moment, $-\langle M_z\rangle B$, lowers the system free energy. For the DDW state, as opposed to superconducting case, there is no screening effect on the magnetic field, the magnetic induction B is homogeneous throughout the system and is close to the external magnetic field H. In the absence of the magnetic field, the pure d-density wave state can be regarded as the equal admixture of the orbital angular moment $L_z = \pm 2$ pairs:

$$W_0(\Theta) = iW_0 \cos(2\Theta) = \frac{iW_0}{2}[\exp(2i\Theta) + \exp(-2i\Theta)] \ . \tag{3}$$

Here we have made an approximation to the order parameter $W_0(\mathbf{k}) \propto \langle c^\dagger_{\mathbf{k+Q},\sigma} c_{\mathbf{k},\sigma}\rangle \propto W_0(\cos k_x a - \cos k_y a)$ by confining the wave vector $\mathbf{k}$ near the Fermi surface and introduced Θ as the 2D azimuthal angle of the Fermi momentum, where $c_{\mathbf{k},\sigma}$ annihilates an electron of spin σ at $\mathbf{k}$, W_0 is the magnitude of the pure d-wave component. In the presence of an external magnetic field, the $L_z = \pm 2$ orbital wave functions becomes unequal and the coefficients for them are shifted linearly with the magnetic field H:

$$\begin{aligned} W_0(\Theta) &\to \frac{iW_0}{2}[(1+\eta B)\exp(2i\Theta) + (1-\eta B)\exp(-2i\Theta)] \\ &= i[W_0(\Theta) + iBW_1(\Theta)] \ , \end{aligned} \tag{4}$$

where $B = H$ and $W_1 \approx \eta \sin(2\Theta)$. Notice that the pure d-density wave order parameter is imaginary while the field generated d'-wave component is real, the relative phase between the two components is still $\pi/2$ in the equilibrium. Microscopically, by focusing on the effect of magnetic field on the order parameter, the system Hamiltonian can be written as:

$$\begin{aligned} H = &\sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + \sum_{\mathbf{k},\sigma} [W_{\mathbf{k}} c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k+Q},\sigma} + \text{H.c.}] \\ &- ig\mu_B B \sum_{\mathbf{k},\sigma} c^\dagger_{\mathbf{k},\sigma} [\sin \mathbf{k}a \times \partial_{\mathbf{k}a}]_z c_{\mathbf{k},\sigma} \ . \end{aligned} \tag{5}$$

Here $\xi_{\mathbf{k}} = -2[\cos k_x a + \cos k_y a] - \mu$ with μ the chemical potential is the single particle energy measured relative to the Fermi energy. The DDW order parameter is given by $W_{\mathbf{k}} = iW_0\varphi_0(\mathbf{k}) + W_1\varphi_1(\mathbf{k})$, where $\varphi_0(\mathbf{k}) = \cos k_x a - \cos k_y a$ and $\varphi_1(\mathbf{k}) = \sin k_x a \sin k_y a$. The amplitude of the d- and d'-wave components $W_{0,1}$ are determined self-consistently:

$$W_0 = \frac{iV_0}{2N} \sum_{\mathbf{k}} \langle c^\dagger_{\mathbf{k+Q},\sigma} c_{\mathbf{k},\sigma}\rangle \varphi_0(\mathbf{k}) \ , \tag{6}$$

and

$$W_1 = -\frac{2V_1}{N} \sum_{\mathbf{k}} \langle c^\dagger_{\mathbf{k+Q},\sigma} c_{\mathbf{k},\sigma}\rangle \varphi_1(\mathbf{k}) \ , \tag{7}$$

where $V_{0,1}$ are, respectively, the d- and d'-channel interaction, N is the number of 2D lattice sites. d-wave component is imaginary due to the equivalence of $\mathbf{Q} = (\pi, \pi)$ and $-\mathbf{Q}$, enforced by the underlying band structure. The notation $[\sin \mathbf{k}a \times \partial_{\mathbf{k}a}]_z$ represents $\sin(k_x a)\partial_{k_y a} - \sin(k_y a)\partial_{k_x a}$. We define $\epsilon_{\mathbf{k}} = -2[\cos k_x a + \cos k_y a]$ so that $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$. For $\mathbf{Q} = (\pi,\pi)$, we have following symmetry properties: $\epsilon_{\mathbf{k+Q}} = -\epsilon_{\mathbf{k}}$, $\varphi_0(\mathbf{k}+\mathbf{Q}) = -\varphi_0(\mathbf{k})$, and $\varphi_1(\mathbf{k}+\mathbf{Q}) = \varphi_1(\mathbf{k})$. In view of the fact that the DDW state breaks the translational symmetry with lattice constant but conserves that by $\sqrt{2}a$ along the diagonals of the square lattice, it is convenient to halve the Brillouin zone, by introducing two kinds of electron operators $c_{\mathbf{k},\sigma}$ and $c_{\mathbf{k+Q},\sigma}$. The pairing of the particles and holes must cause correlations in their relative motions. According to the structure of the Hamitonian and the self-consistency conditions for the DDW order parameter, we can introduce the following Green's functions to describe the correlation:

$$\mathcal{G}_{11}(\mathbf{k},\mathbf{k}';\tau) = -\langle T_\tau[c_{\mathbf{k},\sigma}(\tau) c^\dagger_{\mathbf{k}',\sigma}(0)]\rangle \ , \tag{8}$$

$$\mathcal{G}_{12}(\mathbf{k},\mathbf{k}';\tau) = -\langle T_\tau[c_{\mathbf{k+Q},\sigma}(\tau) c^\dagger_{\mathbf{k}',\sigma}(0)]\rangle \ , \tag{9}$$

$$\mathcal{G}_{21}(\mathbf{k},\mathbf{k}';\tau) = -\langle T_\tau[c_{\mathbf{k},\sigma}(\tau) c^\dagger_{\mathbf{k}'+\mathbf{Q},\sigma}(0)]\rangle \ , \tag{10}$$

$$\mathcal{G}_{22}(\mathbf{k},\mathbf{k}';\tau) = -\langle T_\tau[c_{\mathbf{k+Q},\sigma}(\tau) c^\dagger_{\mathbf{k}'+\mathbf{Q},\sigma}(0)]\rangle \ , \tag{11}$$

where the factor T_τ is a τ-ordering operator as usual, $c_{\mathbf{k},\sigma}(\tau) = e^{H\tau}c_{\mathbf{k},\sigma}e^{-H\tau}$ is the operator in the Heisenberg representation. Given the Hamiltonian Eq. (5), by solving the equation of motion in the approximation up to the first order in the orbital-magnetic field coupling, we obtain the Fourier transform: $\mathcal{G}_{21}(\mathbf{k},\mathbf{k}';i\omega_n) = \mathcal{G}^0_{21}(\mathbf{k},\mathbf{k}';i\omega_n) + \delta\mathcal{G}_{21}(\mathbf{k},\mathbf{k}';i\omega_n)$, where

$$\mathcal{G}^0_{21}(\mathbf{k},\mathbf{k}';i\omega_n) = \frac{(W_\mathbf{k} + W^*_{\mathbf{k+Q}})\delta_{\mathbf{kk}'}}{D(\mathbf{k};i\omega_n)} , \tag{12}$$

and

$$\delta\mathcal{G}_{21}(\mathbf{k},\mathbf{k}';i\omega_n) = \frac{-ig\mu_B B(i\omega_n - \xi_{\mathbf{k+Q}})}{D(\mathbf{k};i\omega_n)} \times[\sin\mathbf{k}a \times \partial_{\mathbf{k}a}]\mathcal{G}^0_{21}(\mathbf{k},\mathbf{k}';i\omega_n) . \tag{13}$$

where $D(\mathbf{k};i\omega_n) = (i\omega_n - E_{\mathbf{k},1})(i\omega_n - E_{\mathbf{k},2})$ with $E_{\mathbf{k},1(2)} = \pm\sqrt{\epsilon^2_\mathbf{k} + |W_\mathbf{k} + W^*_{\mathbf{k+Q}}|^2} - \mu$. We take the ansatz that V_0 is bigger than V_1 [8] such that in the absence of the magnetic field, the d-wave ordering is pure and no secondary phase transition for the appearance of the d' ordering occurs. Therefore, the DDW gap appearing in the $\mathcal{G}^0$ is, $W_\mathbf{k} = iW_0\varphi_0(\mathbf{k})$. As a result, we find:

$$W_1 = -\frac{4V_1}{N}\sum_{\mathbf{k}\in\mathrm{rbz}} \mathrm{Re}[\delta\mathcal{G}_{21}(\mathbf{k},\mathbf{k};\tau=0)]\varphi_2(\mathbf{k}) = \eta B W_0 , \tag{14}$$

where

$$\eta = -\frac{16g\mu_B V_1 k_B T}{N}\sum_{\mathbf{k}\in\mathrm{rbz}}\sum_{\omega_n}\frac{\epsilon_\mathbf{k}\varphi_1^2(\mathbf{k})}{D^2(\mathbf{k};i\omega_n)} \approx \frac{16g\mu_B N(0)V_1}{E_F} , \tag{15}$$

where $N(0)$ is the density of states at the Fermi energy E_F. By taking the Fermi wave length of a few lattice constant a (~ 4Å) and $N(0)|V_1| \sim 0.3$, it is estimated $|W_1/W_0| \approx 10^{-2}$ at $B = 10$T, which makes the amplitude of the induced component $|W_1|$ to be on the order of a few Kelvin.

Equation (14) suggests that the phenomenological Ginzburg-Landau (GL) free energy functional must contain the linear coupling between the original d-density wave order parameter and the field-induced d'-density wave order parameter, $f_{\mathrm{int}} = i\frac{\eta}{2}(iW_0)W_1^* B$ + c.c.. Consequenetly, we can write down the system GL functional of the form:

$$\mathcal{F} = \int d^2r[\frac{\alpha_0}{2}(T - T^0_c)|W_0(\mathbf{r})|^2 + \frac{\beta_0}{4}|W_0(\mathbf{r})|^4 + \frac{K_0}{2}|\nabla(iW_0(\mathbf{r}))|^2 + \frac{K_1}{2}|\nabla W_1(\mathbf{r})|^2 + \frac{\alpha_1}{2}|W_1(\mathbf{r})|^2 + f_{\mathrm{int}}(\mathbf{r})] , \tag{16}$$

where the first two terms describe the instability of the pure d-density wave state, with T^0_c being the transition temperature in the absence of magnetic field. The last two terms represent the energy shift of the d-wave state as a result of the field-induced d'-wave order parameter, where α_1 is positive. Notice that, unlike the superconducting order parameter, the gradient operator on the DDW order parameter is not shifted by the vector potential because the DDW pairs do not carry charge. It follows from Eq. (16) that, as far as the d'-wave component is concerned, the coupling to the magnetic field term [i.e., f_{int}] is linear, while the stiffness term [i.e., the second last term in Eq. (16)] is quadratic. Therefore, at least at the weak field so that W_1 is small, the linear term is dominant. Therefore, the system gains energy by having a nonzero equilibrium value of W_1. By treating W_1 and W_1^* as independent variables, the GL functional $\mathcal{F}$ is minimized by enforcing $\frac{\delta\mathcal{F}}{\delta W_1} = \frac{\delta\mathcal{F}}{\delta W_1^*} = 0$, which leads to

$$W_1 = \frac{\eta B}{\alpha_1}W_0 . \tag{17}$$

Upon substituting the above result into Eq. (16), we find the energy gained by the system with the induced d'-density wave component: $\delta\mathcal{F} = -\int d^2r\eta^2|W_0|^2B^2/2\alpha_1$. Therefore, the transition temperature which is now field dependent, and is renormalized by the magnetic field as: $T_c(B) = T^0_c + \delta T_c(B)$, where $\delta T_c(B) = \eta^2B^2/2\alpha_0\alpha_1$. It then follows that coupling of the magnetic field with the orbital angular momentum shifts the instability of the d-density wave ordering to the high temperature at higher fields. Here we note that, since the particle-hole pairing takes place with the equal spin, the coupling between the magnetic field and the electron spin (i.e., the spin Zeeman coupling) will not depress the induction of d' component in the DDW metal. We do not address here the case of strong field.

Up to now our analysis of the induction of the secondary d' component has been focused on the equilibrium solution. If we assume that this secondary d' order parameter has been created, we can write in general $iW_0 = |W_0|e^{i\phi_0}$ and $W_1 = |W_1|e^{i\phi_1}$, and study the dynamics of the relative phase $\phi = \phi_1 - \phi_0$, which is governed by [9]:

$$\frac{\partial^2\phi}{\partial t^2} = -\rho^{-1}\frac{\delta\mathcal{F}}{\delta\phi} , \tag{18}$$

where $\rho^{-1} \approx N(0)$. With Eq. (16), we find

$$\frac{\partial^2\phi}{\partial t^2} = -\rho^{-1}\eta B|W_0||W_1|\cos\phi - s^2\nabla^2\phi , \tag{19}$$

which leads to the clapping mode with dispersion $\omega^2(B,k) = \omega^2_0(B) + s^2k^2$ with $\omega^2_0(B) = \eta B^2|W_0|^2/\rho$

and $s^2 = |W_0|^2(K_0 + \eta^2 B^2 K_1)/4\rho$. Alternative approach to the clapping mode in d+id' state is presented in [10]. This mode represents the oscillation of the relative phase between the d and d' components of the DDW order parameter, and is tunable by the magnetic field.

We thus have argued that (a) the applied magnetic field can generate the d_{xy} order parameter in the d-density wave metal, whose amplitude is linearly proportional to the field strength, (b) the transition into the $d + id'$-density wave state occurs at a higher transition temperature, and (c) there exists a new clapping mode corresponding to the oscillation of the relative phase between the two components.

3. p + ip' superconductor

Here we consider the case of quasi 2-dimensional p-wave superconductor in an external field. We identify here $|\Psi_0\rangle \propto p_x$ and $|\Psi_1\rangle \propto p_y$. Assume that the zero field state has real order parameter that transforms as $|\Psi_0\rangle \propto p_x$. We argue then that external magnetic field will induce secondary component $|\Psi_1\rangle = ip_y$. The state $|\Psi_0\rangle + |\Psi_1\rangle$ will have a finite magnetic moment $< M_z >$ that can couple to magnetic field. The free energy term driving the secondary component is again given by Eq.(16) and we find $|\Psi_1\rangle \propto B|\Psi_0\rangle$ in case of p-wave superconductor. Here we do not address the spin part of the order parameter that can also couple to magnetic field. The results for the clapping mode are similar to the case of d-wave superconductor. We note that the similar clapping mode for the p-wave state that violates time reversal in zero field, e.g. $p_x + ip_y$, was considered in [11].

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