Magnetic equation of state for ^{3}He in thin ^{3}He–^{4}He superfluid films

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Abstract

We examine the thermodynamics of a two–dimensional Fermi gas model describing adsorbed ^{3}He in a thin superfluid ^{4}He film. We show that the magnetic steps seen in experiment are a simple manifestation of the $T = 0K$ equation of state and that the thermal stability of the steps is determined by the larger of $\Delta\epsilon/2$, one–half of the level spacing, and $\mu_m\mathcal{H}_0$, the magnetic energy. We derive the conditions under which there exist points in the phase space (termed invariant points) through which all low-temperature isotherms pass exponentially close. The calculated invariant points for the magnetic susceptibility versus coverage are in good agreement with experiment. We calculate the magnetic susceptibility and show that it should exhibit a step-like structure versus ^{3}He coverage similar to that of the magnetization.

Key words: ^{3}He –^{4}He mixture films; two-dimensional fermions; thermodynamics

1. Introduction

In 1989 Higley, Sprague and Hallock [1,2] discovered steps in the magnetic equation of state (magnetization isotherms versus coverage) for thin ^{3}He –^{4}He mixture films. The steps are direct evidence of the existence of a set of discrete, transverse ^{3}He states due to the external substrate. These states were first predicted and examined by Gasparini and co-workers [3] in a pioneering series of heat capacity experiments. In both the Hallock and Gasparini experiments, the solid substrate was Nuclepore, a polycarbonate material threaded by roughly cylindrical passages of nominal diameter 2000 Å.

We consider a system of $\mathcal{N}$ ^{3}He atoms in an area $\mathcal{A}$. The spin-$\frac{1}{2}$ atoms have magnetic moment μ_m and are subject to an applied magnetic field $\mathcal{H}_0$. The spin state will be labeled by the index $s = \pm$ where we can arbitrarily choose $s = +$ to represent the low energy configuration. The ^{3}He atoms are adsorbed onto a film of ^{4}He which is itself adsorbed onto a solid substrate. All information in the ^{3}He system concerning the ^{4}He film and the substrate is contained in a set of transverse single–particle states whose energy levels are denoted $\{\epsilon^0_\alpha\}$. In the numerical work to be described below, we shall use $\mathcal{H}_0$= 2 T. The magnetic energy, $2\mu_m\mathcal{H}_0$, is approximately 30 mK. Our model has two transverse states with energies $\{\epsilon^0_0 = 0, \epsilon^0_1 = 1.8\ K\}$ and an effective mass, $m^* = 1.38m_3$. These values were chosen to agree with the NMR experiments of Ref. [2].

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2. Thermodynamics

For fermions in two-dimensions, the density and magnetization for particles in level $\{\alpha, s\}$ at temperature $\mathcal{T}$, $\beta = 1/k_B\mathcal{T}$, are given by:

$$\overline{n} = (1/2w_\ell)\sum_{\alpha,s}\ln(1 + \Lambda_{\alpha,s}), \qquad (1)$$

where $w_\ell = \beta\epsilon_{\mathrm{F}\ell}$, and

$$\overline{m} = (1/2x)\sum_{\alpha}\ln\left(\frac{1+\Lambda_{\alpha+}}{1+\Lambda_{\alpha-}}\right), \qquad (2)$$

where $x = \beta\mu_m\mathcal{H}_0$ and $\Lambda_{\alpha,s} = \exp[\beta(\mu-\epsilon^0_\alpha+\mu_m\mathcal{H}_0 s)]$. The bars denote dimensionless quantities. Densities are

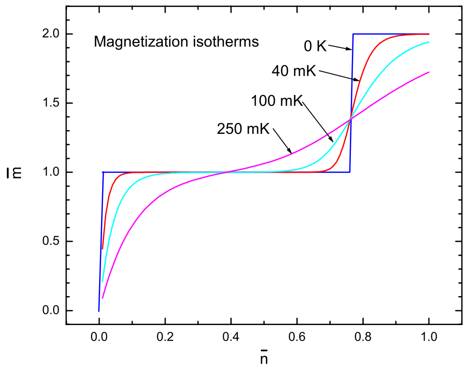

Fig. 1. Magnetization, $\overline{m}$, as a function of coverage, $\overline{n}$, for temperatures = 0 K, 40 mK, 100 mK, 250 mK.

measured in units of one complete ^{3}He monolayer, $n_\ell = 0.0647$ Å^{-2}. Magnetizations are measured in units of $\mathcal{M}_0 = m^* \mu_m^2 \mathcal{H}_0 \mathcal{A}/\pi\hbar^2$, the value of the magnetization at the first step.

In Figure 1, we show the magnetic equation of state (isotherms of magnetization versus coverage). The evolution of the step structure out of the zero temperature results is clear. The temperatures chosen for the isotherms match those in Ref [2].

In the low temperature limit, with $\Delta\epsilon \gg \mu_m \mathcal{H}_0$, we find for the slope at $\overline{m} = 1$,

$$(d\overline{m}/d\overline{n})_{\overline{m}=1} \approx (1/\overline{n}_{0L}) \exp[-w/2 + x], \tag{3}$$

where $w = \beta\Delta\epsilon$, $\overline{n}_{0L} = \mu_m \mathcal{H}_0/\epsilon_{\mathrm{F}\ell}$ and $\epsilon_{\mathrm{F}\ell} = \hbar^2 \pi n_\ell / m^*$ is the Fermi energy of a completed monolayer. Thus, the *larger* of $\mu_m \mathcal{H}_0$ or $\Delta\epsilon/2$ determines the region of temperature stability for the step. In the case of Fig. 1, $\Delta\epsilon/2 = 0.9$ K and $\mu_m \mathcal{H}_0 = 16$ mK.

The points in the thermodynamic phase space that all isotherms seem to pass through will be called invariant points. On Fig. 1, there are two non–trivial points located at $(\overline{n}, \overline{m}) = (\overline{n}_{\mathrm{onset}}/2, 1), (\overline{n}_{\mathrm{onset}}, 4/3)$. [We note $\overline{n}_{\mathrm{onset}} = \Delta\epsilon/\epsilon_{\mathrm{F}\ell} = 0.761$.] It is straightforward to show that all isotherms pass exponentially close to these points at low enough temperature. For example, for the invariant point at $\overline{m} = 1$, we find

$$\overline{n} \approx (1/2)\, \overline{n}_{\mathrm{onset}} + O(e^{-w/2}). \tag{4}$$

Thus for temperature less than $\Delta\epsilon/2$ all magnetization isotherms come exponentially close to this invariant point. These results are in qualitative agreement with experiment. [2]

We also consider the magnetic susceptibility, χ, at fixed system size. In the limit, $\mathcal{H}_0 \to 0$, we obtain:

$$(\chi/\chi_0)_{\mathcal{H}_0=0} = \sum_\alpha \left(\Lambda_\alpha/(1+\Lambda_\alpha)\right), \tag{5}$$

where $\Lambda_\alpha = \exp\beta(\mu - \epsilon_\alpha^0)$ and $\chi_0 = \mathcal{M}_0/\mathcal{H}_0$ is the Pauli susceptibility. In Fig 2 we show the magnetic susceptibility at temperatures 0, 40, 100 and 250 mK. This figure shows clearly that the magnetic susceptibility exhibits steps at integer values similar to the magnetization steps of Fig. 1. Neither existing susceptibility measurements nor first principles calculations have gone to high enough coverage to see the predicted steps. [1]

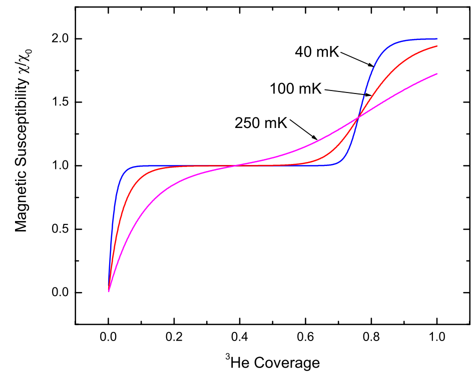

Fig. 2. Magnetic susceptibility in units of the Pauli susceptibility as a function of ^{3}He coverage at temperatures 40, 100, and 250 mK. At $\mathcal{T} = 0$ K the susceptibility would be horizontal steps at $\chi/\chi_0 = 1, 2$ with a vertical ramp at $\overline{n} = 0.77$.

The magnetic susceptibility versus coverage has two non-trivial invariant points that can be analyzed exactly as the invariant points in Fig. 1.

3. Conclusion

In this paper, we examined the step structure and invariant points in the magnetization and magnetic susceptibility for ^{3}He in thin ^{3}He –^{4}He superfluid films.

See Ref. [4] for a detailed description of the ground–state and also a discussion of the behavior of the pressure, the chemical potential, the heat capacities at fixed area and pressure, the thermal expansion coefficient and the isothermal compressibility.

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