Multiple components of the order parameter induced around the $d_{x^2-y^2}$-wave vortex core

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Abstract

On the basis of the Bogoliubov-de Gennes theory for the two-dimensional extended Hubbard model, the vortex structure of $d_{x^2-y^2}$-wave superconductors is analyzed. Multiple components of the order parameter are induced around the $d_{x^2-y^2}$-wave vortex core. We study the spatial structure of the induced $p_x \pm ip_y$ and extended s-wave-wave components. We also study the case when the antiferromagnetism(checkerboard or stripe) is induced at the vortex core. There, spin-triplet $d_{x^2-y^2}$-wave component is also induced.

Key words: Bogoliubov-de Gennes theory; vortex lattice; order parameter

Much attention has been focused on vortex physics in high-T$_c$ superconductivity. In the mixed state, d-wave order parameter is suppressed at the vortex core. The spatial variation of the order parameter induces the other symmetry of the order parameter. In this paper, we study the multiple components of the order parameter induced around the $d_{x^2-y^2}$-wave vortex core on the basis of Bogoliubov de-Gennes theory.

We begin with the extended Hubbard model on a two-dimensional square lattice, and introduce the mean field $n_{i,\sigma} = \langle a^\dagger_{i,\sigma} a_{i,\sigma}\rangle$ at the i-site, where σ is a spin index and $i = (i_x, i_y)$. We assume a pairing interaction V between nearest-neighbor (NN) sites. This type of pairing interaction gives d-wave superconductivity[1,2]. Thus, the mean-field Hamiltonian under H is given by

$$H = -\sum_{i,j,\sigma} \tilde{t}_{i,j} a^\dagger_{i,\sigma} a_{j,\sigma} + U \sum_{i,\sigma} n_{i,-\sigma} a^\dagger_{i,\sigma} a_{i,\sigma} + V \sum_{\hat{e},i,\sigma} (\Delta^\dagger_{\hat{e},i,\sigma} a_{i,-\sigma} a_{i+\hat{e},\sigma} + \Delta_{\hat{e},i,\sigma} a^\dagger_{i,\sigma} a^\dagger_{i+\hat{e},-\sigma})$$

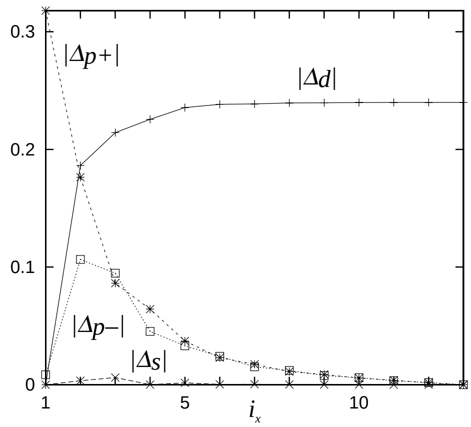

Fig. 1. The amplitudes of the order parameter Δ_d, Δ_s, Δ_{p+} and Δ_{p-} along the x-direction. The vortex core locates at $i_x = 1$. $U/t = 0.0$

where $a^\dagger_{i,\sigma}$ ($a_{i,\sigma}$) is a creation (annihilation) operator, and $i + \hat{e}$ represents the NN site ($\hat{e} = \pm\hat{x}, \pm\hat{y}$). The transfer integral is expressed as

$$\tilde{t}_{i,j} = t_{i,j} \exp[\mathrm{i}\frac{\pi}{\phi_0} \int_{\mathbf{r}_i}^{\mathbf{r}_j} \mathbf{A}(\mathbf{r}) \cdot \mathrm{d}\mathbf{r}],$$

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with the vector potential $\mathbf{A}(\mathbf{r}) = \frac{1}{2}\mathbf{H} \times \mathbf{r}$ in the symmetric gauge, and the flux quantum ϕ_0. For the NN pairs (i, j), $t_{i,j} = t$. For the next-NN pairs situated on a diagonal position on the square lattice, $t_{i,j} = t'$. For the third-NN pairs, which are situated along the NN bond direction, $t_{i,j} = t''$. To reproduce the Fermi surface topology of cuprates, we set $t' = -0.12t$ and $t'' = 0.08t$. We consider the pairing interaction $V = -2.0t$. The essential results of this paper do not significantly depend on the choice of these parameter values.

In terms of the eigen-energy E_α and the wave functions $u_\alpha(\mathbf{r}_i)$, $v_\alpha(\mathbf{r}_i)$ at the i-site, the BdG equation is given by

$$\sum_j \begin{pmatrix} K_{\uparrow,i,j} & D_{i,j} \\ D^\dagger_{i,j} & -K^*_{\downarrow,i,j} \end{pmatrix} \begin{pmatrix} u_\alpha(\mathbf{r}_j) \\ v_\alpha(\mathbf{r}_j) \end{pmatrix} = E_\alpha \begin{pmatrix} u_\alpha(\mathbf{r}_i) \\ v_\alpha(\mathbf{r}_i) \end{pmatrix},$$

where $K_{\sigma,i,j} = -\tilde{t}_{i,j} + \delta_{i,j}(Un_{i,-\sigma} - \mu)$, $D_{i,j} = V\sum_{\hat{e}} \Delta_{i,j}\delta_{j,i+\hat{e}}$ and α is an index of the eigenstate[1,2]. The self-consistent condition for the pair potential and the number density is given by $\Delta_{i,j} = \langle a_{j,\downarrow}a_{i,\uparrow}\rangle = \sum_\alpha u_\alpha(\mathbf{r}_i)v^*_\alpha(\mathbf{r}_j)f(E_\alpha)$, $n_{i,\uparrow} = \langle a^\dagger_{i,\uparrow}a_{i,\uparrow}\rangle = \sum_\alpha |u_\alpha(\mathbf{r}_i)|^2 f(E_\alpha)$, $n_{i,\downarrow} = \langle a^\dagger_{i,\downarrow}a_{i,\downarrow}\rangle = \sum_\alpha |v_\alpha(\mathbf{r}_i)|^2(1 - f(E_\alpha))$. The charge density $n_i = n_{i,\uparrow} + n_{i,\downarrow}$, the spin density $S_{z,i} = \frac{1}{2}(n_{i,\uparrow} - n_{i,\downarrow})$ and the staggered magnetization $M_i = S_{z,i}(-1)^{i_x+i_y}$. The d-wave order parameter at site i is

$$\Delta_{d,i} = (\Delta_{\hat{x},i} + \Delta_{-\hat{x},i} - \Delta_{\hat{y},i} - \Delta_{-\hat{y},i})/4$$

with $\Delta_{\hat{e},i} = \bar{\Delta}_{i,i+\hat{e}} \exp[\mathrm{i}\frac{\pi}{\phi_0}\int_{\mathbf{r}_i}^{(\mathbf{r}_i+\mathbf{r}_{i+\hat{e}})/2} \mathbf{A}(\mathbf{r}) \cdot \mathrm{d}\mathbf{r}]$, where $\bar{\Delta}_{i,i+\hat{e}} = \langle a_{i+\hat{e},\downarrow}a_{i,\uparrow}\rangle - \langle a_{i+\hat{e},\uparrow}a_{i,\downarrow}\rangle$.
The extended-s-wave and $p_x \pm p_y$-wave order parameters at site i are

$$\Delta_{s,i} = (\Delta_{\hat{x},i} + \Delta_{-\hat{x},i} + \Delta_{\hat{y},i} + \Delta_{-\hat{y},i})/4,$$
$$\Delta_{p+,i} = (\Delta_{\hat{x},i} - \Delta_{-\hat{x},i} + \mathrm{i}\Delta_{\hat{y},i} - \mathrm{i}\Delta_{-\hat{y},i})/4,$$
$$\Delta_{p-,i} = (\Delta_{\hat{x},i} - \Delta_{-\hat{x},i} - \mathrm{i}\Delta_{\hat{y},i} + \mathrm{i}\Delta_{-\hat{y},i})/4.$$

The triplet-d-wave order parameter at site i is

$$\Delta^{triplet}_{d,i} = (\Delta^{triplet}_{\hat{x},i} + \Delta^{triplet}_{-\hat{x},i} - \Delta^{triplet}_{\hat{y},i} - \Delta^{triplet}_{-\hat{y},i})/4$$

where $\bar{\Delta}^{triplet}_{i,i+\hat{e}} = \langle a_{i+\hat{e},\downarrow}a_{i,\uparrow}\rangle + \langle a_{i+\hat{e},\uparrow}a_{i,\downarrow}\rangle$.

We typically consider the case of a unit cell with 24×24 sites, where two vortices are accommodated. The spatially averaged hole density is set to $n_\mathrm{h} = 1 - \overline{n_i} \sim \frac{1}{8}$ by tuning the chemical potential μ. By introducing the quasimomentum of the magnetic Bloch state, we obtain the wave function under the periodic boundary condition whose region covers many unit cells.

Figure 1 shows the cross section of the order parameter's amplitude. The d-wave component $|\Delta_d| = 0$ at the vortex core and recover its amplitude continuously

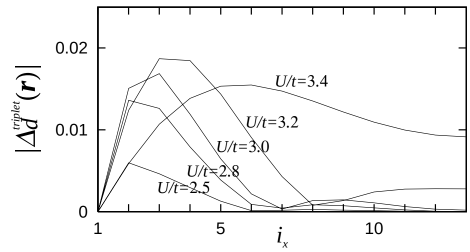

Fig. 2. The amplitede of the triplet-d-wave component around the core along the x-direction, when magnetization appear at the vortex core. The stripe state apears at U/t=3.4. The vortex core locates at $i_x = 1$.

toward outside the core. The argument of the d-wave component has +1-winding at the vortex core. Other components are induced around the core. s-wave and $p_x - ip_y$-wave components are zero at the core, and $p_x + ip_y$-wave component has finite value at the core. The argument of $p_x - ip_y$-wave component has +2-winding at the vortex core, −1-winding at the middle of nearest vortices and +1-winding at the middle of next nearest vortices. The argument of $p_x + ip_y$-wave component has +1-winding at the middle of nearest vortices and −2-winding at the middle of next nearest vortices.

As increasing the repulsive interaction U, the magnetization arises around the vortex core. At the range of $U/t > 2.5$, checkerboard type weak modulation in $M(\mathbf{r})$ appears, and at the range of $U/t > 3.4$, the stripe state which is modulated by vortices[3] appears. In these cases, the triplet-d-wave component appears around the core. Figure 2 shows the cross section of the triplet-d-wave component's amplitude. The maximum point moves to outside the core by increasing U. At $U/t = 3.4$, stripe state appears. As U increase, $p_x \pm ip_y$-wave amplitudes become small, and extended-s-wave and triplet-d-wave amplitudes become large. Additionally, in the stripe state, extended-s-wave argument has clearly −1-winding at the core.

In summary, we study the multiple components of the order parameter induced around the $d_{x^2-y^2}$-wave vortex core by solving the Bogoliubov de-Gennes theory. The spatial variation of the order parameter induces other symmetry of the order parameter. As the magnetization appears around the vortex core, the spin-triplet $d_{x^2-y^2}$-wave component is also induced.

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