Melting Rate of U2D2 Solid ^{3}He

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Abstract

We measured the growth coefficient of nuclear-ordered solid ^{3}He in a low magnetic field of 0.33 T and in a temperature range from 0.5 to 0.9 mK. The growth coefficient in melting process is strongly dependent on temperature and increases as T^{-n} when temperatures decreased, where n is approximately 5.

We evaluated the melting rate for a rough surface using two models and compared these results with the experiment. We derived the intrinsic growth coefficient limited by the energy dissipation due to magnon scattering at the interface. The other model is the extrinsic one in which the thermal impedance of liquid and Kapitza resistance at the solid-liquid boundary limit the melting rate. We concluded that the thermal impedance limited the melting rate.

Key words: Crystal Growth; Crystal Melting; Solid ^{3}He

The unique features of ^{4}He at low temperature, where the latent heat for solidification is very small and heat and mass transport in superfluid occur smoothly, have enabled us to study thoroughly the crystal growth mechanism and equilibrium shape of crystal. In the case of ^{3}He, these conditions are met below 1 mK, where the liquid is in the superfluid B phase and the solid is in the nuclear-ordered U2D2 phase.

We investigated the growth and melting mechanisms and their temperature dependence in a low magnetic field of 0.33 T and in a temperature range between 0.5 mK and the nuclear-ordered temperature $T_{\mathrm{N}} = 0.87$ mK. The experimental procedure and the results were reported in a previous paper [1]. A crystal was grown and melted in the rectangular part of a compressible cell with a cross-sectional area of $2 \times 2\,\mathrm{mm}^2$. The effective length from the crystal surface to the heat exchanger was about 54 mm. We measured the effective growth coefficient defined as follows,

$$\kappa_{\mathrm{eff}} = v/\Delta\mu, \tag{1}$$

where v is the velocity of the interface and $\Delta\mu$ is the chemical potential difference per unit mass at the crystal surface. The growth coefficient in the melting process is much larger than that in the growth process and strongly depended on temperature. In the previous paper we analyzed the growth mechanism and attributed it to the spiral growth of the faceted crystal. In this paper, we discuss the melting mechanism on a rough surface and compare it with our experimental results.

During the measurement of the melting process, we kept v to less than 10 μm/s, which is much less than the pair-breaking critical velocity of superfluid and the spin wave velocity of U2D2 solid. This condition guarantees fast mass and spin transport involved by melting.

The motion of the interface is accompanied by surface scattering of thermal excitations both in liquid and in solid. The growth coefficient was calculated for the ^{4}He case in Refs. [2,3] and for ^{3}He in Ref. [4]. In the ^{3}He system the most effective excitation is thermal magnon in solid. Assuming that the energy spectrum for magnon is isotropic, $\epsilon = cp$ ($p = |\boldsymbol{p}|$, $\boldsymbol{p}$ is the momentum, and c is the spin wave velocity), and using a similar formula as Refs. [2–4], we obtain the melting rate κ_{int} as,

$$\frac{1}{\kappa_{\mathrm{int}}} = \frac{\pi^2\hbar}{15\rho_{\mathrm{s}}}\left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^4, \tag{2}$$

where ρ_{s} is the density of solid and we sum up two

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spin acoustic wave modes in U2D2 solid and neglect the optical mode.

Moving interface also releases latent heat, so that the temperature at the interface is not constant while melting. The melting rate, $\kappa_{\rm eff}$, is not only controlled by $\kappa_{\rm int}$, but also by extrinsic factors of liquid thermal resistance, $Z_{\rm l}$, Kapitza boundary resistance, $R_{\rm K}$, and solid thermal resistance, $Z_{\rm s}$ (see Ref. [5] Eq. (6)). In our experimental set-up no heat can be absorbed through the solid so that $Z_{\rm s} \gg Z_{\rm l}, R_{\rm K}$. The effective melting rate is described as the sum of three terms,

$$\begin{aligned}\frac{1}{\kappa_{\rm eff}} &= \frac{1}{\kappa_{\rm int}} + \frac{\rho_{\rm s}}{T}L^2 Z_{\rm l} + \frac{\rho_{\rm s}}{T}(TS_{\rm s}-\lambda)^2 R_{\rm K} \\ &\equiv \frac{1}{\kappa_{\rm int}} + \frac{1}{\kappa_{\rm liq}} + \frac{1}{\kappa_{\rm R}}, \qquad (3)\end{aligned}$$

where $L = T(S_{\rm s} - S_{\rm l})$ is latent heat, $S_{\rm s,l}$ are entropy of solid and liquid, respectively. The parameter λ is the fraction of L released to solid and can be a value between $TS_{\rm s}$ and 0.

We calculated the liquid thermal resistance $Z_{\rm l}$ in two regimes. In the ballistic region, where the quasi-particle mean free path is much longer than the channel size, the thermal resistivity was measured by Feng et al. at melting pressure [6]. Considering geometrical difference of sample cells, we obtained thermal conductivity k for our sample cell as,

$$k^{-1} = 4.7\,T\exp\left(\Delta/k_{\rm B}T\right) \quad [\mathrm{Km/W}]. \qquad (4)$$

In the diffusive regime, Wellard et al. [7] made a heat-pulse measurement at 10, 20 and 29 bar. From their data, kT does not depend on temperature very much in the superfluid and we used $kT = 7.3 \times 10^{-5}$ W/m.

We express $\kappa_{\rm liq}$ as a function of temperature in ballistic and diffusive regions respectively,

$$\text{Ballistic}: \ {\kappa_{\rm liq}}^{-1} = 5.3\,T^8\exp\left(4.8/T\right) \quad [\mathrm{m/s}], \qquad (5)$$

$$\text{Diffusive}: \ {\kappa_{\rm liq}}^{-1} = 2.0 \times 10^4\,T^8 \quad [\mathrm{m/s}], \qquad (6)$$

where, the unit of T is mK.

From Ref. [8], the mean free path in the superfluid B phase is $0.4\exp\left(\Delta/k_{\rm B}T\right)$ μm. Comparing the mean free path to our sample cell size, 2 mm, the thermal transport mechanism changes from ballistic to diffusive at $T \sim 0.56$ mK. The temperature range where we measured the melting rate is in the diffusive regime.

To evaluate $\kappa_{\rm R}$, we consider its maximum contribution. When the Kapitza resistance measured by Feng et al. [6] is used, the upper limit of $1/\kappa_{\rm R}$ for $\lambda = 0$ is,

$${\kappa_{\rm R}}^{-1} \leq 3.8 \times 10^{-2}\,T^3\exp\left(4.2/T\right) \quad [\mathrm{m/s}]. \qquad (7)$$

Here also the unit of T is mK.

In Fig. 1 we compare the experimental data with the calculated value from the Eqs. (2), (5), (6) and (7). The effects of Kapitza resistance and intrinsic mechanism are very small and not important in our measurement. The contribution of the thermal impedance of liquid, Eq. (6), is comparable to the observed value. We conclude that our measured temperature dependence of the melting rate is determined by $\kappa_{\rm liq}$ in the diffusive regime. The measured value κ changes approximately as T^{-5}, whereas $\kappa_{\rm liq}$ changes as T^{-8}. It seems that this disagreement is caused by uncertainty of the thermal conductivity. Melting rates of U2D2 solid are also measured in Refs. [9–11] and the results agree with our data in the order of magnitude. It is noted that the melting rates measured by these groups seem to be limited by thermal impedance in the liquid, although the cell geometries may differ from each other. Extension of the measurement in the ballistic region is needed in order to investigate whether the melting rate decreases as temperature decreases.

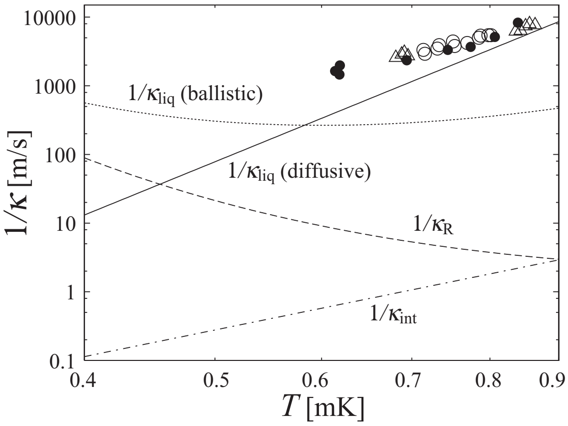

Fig. 1. Temperature dependence of $1/\kappa$: ○,●,△ are experimental data for crystals with different orientations.

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