

# Current and Shot Noise in Superconducting Junctions with a Quantum Dot in the Kondo Regime.

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## Abstract

Transport through an Anderson impurity in the Kondo regime situated between two superconductors is considered. Here the analysis is performed within the slave boson mean field approximation. The new physics follows due to multiple Andreev reflections at the boundary between the dot and superconductor. The important new parameter which enters the transport characteristics is the ratio of the Kondo temperature to the superconducting gap. The current, shot noise power are displayed versus the applied bias voltage in the subgap region and found to be strongly dependent on this ratio. In particular, the  $I - V$  curve exposes an excess current in the limit of high Kondo temperature.

*Key words:* Kondo effect ; Fano factor; excess current;

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## 1. Background

A number of recently developed experimental techniques allow for detailed investigations of electronic transport through atomic-size metallic conductors. Usually, transport properties of such systems are strongly affected by Coulomb interactions. Novel physical effects emerge if electrodes of an atomic-size contact become superconducting. In that case the mechanism of multiple Andreev reflections (MAR) plays a dominant role being responsible for both dc Josephson effect and for dissipative currents at subgap voltages. Further possibilities for experimental investigation of an interplay between MAR and Coulomb effects in systems with few conducting channels are provided by recently fabricated superconducting junctions with a weak link formed by a carbon nanotube[1].

In the present work we study the limit of sufficiently *high* Kondo temperatures. This case is relevant for a junctions with carbon nanotubes [1] where the Kondo effect with rather high Kondo temperature

$T_K \sim 1.6K$  was recently observed [2]. In the Kondo regime a ballistic-like channel opens up inside the dot. Hence, an interplay between MAR and the Kondo resonance is expected to yield, e.g. an excess current on the  $I - V$  curve similarly to the case of noninteracting ballistic junctions. At very large values of  $T_K$  and in the low voltage limit this current should approach the noninteracting result  $I_{AR} = 4e\Delta/h$ . Analogously, the shot noise power is expected to display a pronounced maximum at  $V = 0$  and should decay as  $1/V$  at small bias as is familiar in the standard noninteracting *SNS* junction. Below we will present a quantitative analysis which fully supports this qualitative physical picture.

## 2. The Model

The system is represented by two electrodes (*L* and *R*), weakly coupled to the point-like Anderson impurity located at the origin. The system dynamics is governed by the Hamiltonian

$$H = H_L + H_R + H_d + H_t + H_c, \quad (1)$$

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in which  $H_{L,R}$  are the BCS Hamiltonians of the electrodes which depend on the electron field operators  $\psi_{j\sigma}(\mathbf{0}, t)$ , where  $j = L, R$  represents left-right lead, and  $\sigma$  is the spin index. The dot is described as a single level Anderson impurity  $A$  with energy  $\epsilon_0 < 0$  and Hubbard repulsion parameter  $U$ . In the Kondo regime of interest here we set  $U \rightarrow \infty$  and assume  $|\epsilon_0|$  to be larger than any other energy scale except for  $U$ . In this case it is convenient to express the dot and the tunneling Hamiltonians  $H_d$  and  $H_t$  via slave boson (operators  $b, b^\dagger$ ) and slave fermion (operators  $c, c^\dagger$ ) auxiliary fields. Explicitly,  $H_d = \epsilon_0 \sum_\sigma c_\sigma^\dagger c_\sigma$  and  $H_t = \tilde{t} \sum_{j\sigma} c_\sigma^\dagger b \psi_{j\sigma}(\mathbf{0}, t) + \text{h.c.}$ , where  $\tilde{t}$  is the tunneling amplitude. Finally, the Hamiltonian of the system must also include a term which prevents double occupancy in the limit  $U \rightarrow \infty$ . This term reads  $H_c = \lambda (\sum_\sigma c_\sigma^\dagger c_\sigma + b^\dagger b - 1)$ , where  $\lambda$  is a Lagrange multiplier. After integrating out the leads variables we reduced the problem to the dynamically interacting one impurity problem. In order to describe the Kondo regime we treat the slave boson fields within the dynamical mean field approximation (DMFA)[3]. In the DMFA we replace the bose operators by their expectation values. These mean field values as well as the Lagrange multiplier which, in fact, fixes the position of renormalized impurity level are determined by minimization of the effective action of the system. Actually, the one from two equations that follow from such minimization reflects the condition which prevents double occupancy in the limit  $U \rightarrow \infty$ .

The MFSBA is known to encode the Kondo Fermi-liquid behavior at low temperatures. An important parameter here is the ratio between the Kondo temperature and the superconducting gap  $t_K \equiv T_K^0/\Delta$ . For  $t_K > 1$  a Fermi liquid behavior is expected. Quantitatively, the MFSBA is reliable only for sufficiently large values of  $t_K$ . It is worth noting here that the applied bias voltage  $V$  also attenuates the Kondo resonance and lowers  $T_K$ . Hence, for the reliability of the MFSBA in non-equilibrium situations, both  $\Delta$  and  $eV$  should not exceed the Kondo temperature. Attention below is mainly focused on the subgap voltage regime  $eV \leq \Delta$  in which case  $t_K$  appears to be the only relevant parameter.

We begin with the calculations the current as function of applied voltage. For sufficiently large  $t_K$  we expect strong Kondo resonance and the  $I - V$  curve is anticipated to resemble that of purely ballistic junctions without interaction. This agreement is supported by our numerical analysis for large  $t_K = 100$  which corresponds to the unitary, pure ballistic case (see Fig.1). Calculation of the current is carried out also for  $t_K = 5$  and 1.6. We notice that while for  $t_K = 5$  the imprint of ballistic behavior is still remains (finite excess current), the I-V characteristic noticeable deviates from the unitary limit. For  $t_K = 1.6$  the competition be-

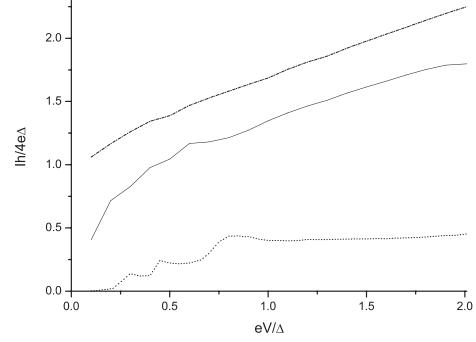


Fig. 1. The averaged current  $I$  (in units of  $4e\Delta/h$ ) versus the bias  $V$  (in units of  $\Delta/e$ ) for an SAS junction at sub-gap voltages with  $\Gamma/T_K^0 = 200$ . Curves from top to down (dashed-dotted, solid and dotted) correspond respectively to  $t_K = 100, t_K = 5$  and  $t_K = 1.6$ .

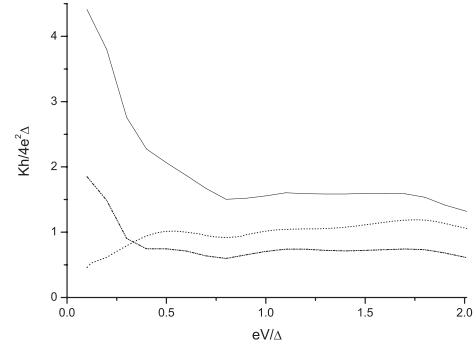


Fig. 2. The shot noise power  $K$  (in units of  $2e^2\Delta/h$ ) as a function of  $V$  (in units of  $\Delta/e$ ) for an SAS junction. The parameters and notations are the same as in Fig.1 (the top and lowest curves correspond to  $t_K = 5$  and  $t_K = 1.6$ , respectively)

tween gap-related suppression of the Kondo effect and effective transparency of the junction becomes essential leading to further decreasing of the current.

The current shot noise is defined by current-current correlation function. The shot noise power spectrum versus applied voltage is displayed in Fig.2 for the same set of parameters as the  $I - V$  curves. In the case  $t_K \gg 1$  the curve turns to  $1/V$  dependence at low voltages (ballistic regime).

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