Calculation of Electron Spin Polarization for Fractional Quantum Hall States

Shosuke Sasaki [a,1]

[a] *Shizuoka Institute of Science and Technology, Toyosawa 2200-2, Fukuroi 437-8555, Japan*

Abstract

Many plateaus with various widths appear in the experimental data of magnetic dependence of the electron spin polarization in the fractional quantum Hall effect. In the Landau gauge, single electron orbitals have the shape of equal spaced parallel lines. The many electron state is constructed by a specific electron-configuration so as to have minimum value of the classical Coulomb energy. The residual Coulomb interaction produces electron spin exchange interactions. We additionally consider the spin-Peierls effect. When we modulate the intervals between the orbitals, a small energy gap appears. Consequently, the total energy caused by this modulation is smaller than one in the non-modulation case, and therefore this modulation really appears. We have obtained all the eigenenergies for the Hamiltonian with spin exchange interactions under this modulation. Then, the calculated curve of the spin polarization shows both narrow plateaus and wide plateaus, and is in good agreement with the experimental data.

Key words: lambda transition; superfluidity; helium4; phase diagram

1. Spin exchange interaction

Kukushkin, Klitzing, and Eberl have measured the electron spin polarization in the fractional quantum Hall effect (FQHE) for twelve filling factors [1]. Their results show various types of behaviors in the magnetic dependence of the spin polarization. We explain these behaviors by solving eigen equations in the present paper. In the integral quantum Hall effect, Laughlin et al have used Landau gauge, and have clarified the quantization of Hall resistance [2]. Tao and Thouless [3] have adopted these states in FQHE. But their ground states are degenerate, so it is difficult to diagonalize the Coulomb interactions between electrons. In the previous paper, we have divided the total Hamiltonian $H_{\rm T}$ into two parts as $H_{\rm T} = H_{\rm D} + H_{\rm I}$, where the new 0-th order Hamiltonian $H_{\rm D}$ includes the classical Coulomb energy. Then, our 0-th order ground state possesses an electron configuration with a minimum eigenvalue of $H_{\rm D}$, and therefore is not degenerate (see reference [4]). For the filling factor ν=2/3 as an example, the electron configuration is the repetition of the sequence (filled, filled, empty) as is shown in figure (1).

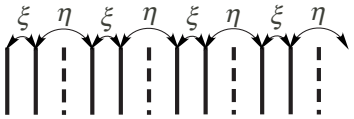

Fig. 1. Interactions for the filling factor of 2/3 (the dashed lines show empty orbitals)

This electron-configuration is the most uniform distribution of electrons for ν=2/3, and therefore has the minimum classical Coulomb energy. We take account of the residual Coulomb interactions, which produce the spin exchange interaction. The spin-exchange interaction has the form as $(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+)$ where σ^+ is the transformation operator from a down-spin state to an up-spin state, and σ^- is the Hermitian conjugate operator of σ^+. When we consider the interactions be-

[1] Corresponding author. Present address: Shizuoka Institute of Science and Technology, Toyosawa 2200-2, Fukuroi 437-8555, Japan E-mail: sasaki@ns.sist.ac.jp

tween the nearest-neighbor electrons only, we obtain an approximate Hamiltonian H as

$$H = \sum_{i=1,2,3,\cdots} [\xi(\sigma^+_{2i-1}\sigma^-_{2i} + \sigma^-_{2i-1}\sigma^+_{2i}) + \eta(\sigma^+_{2i}\sigma^-_{2i+1} + \sigma^-_{2i}\sigma^+_{2i+1})] + \sum_{i=1,2,3,\cdots} \frac{\mu_B g}{\mu_0} B \frac{1}{2}\sigma^z_i \quad (1)$$

where ξ is the coupling constant between two electrons in the nearest orbital-pair, and η is the coupling constant between two electrons in the second nearest orbital-pair, g is the g-factor, B is the magnetic field strength, and μ_B is the Bohr magneton. The eigenvalue problem for Hamiltonian (1) can be solved by the method of Reference [5].

2. Spin Peierls Effect and spin polarization

We consider the mechanism of spin-Peierls transition [6]. That is to say, modulation of intervals between the orbitals creates two extra-energies, namely, 1) energy-increasing in the classical Coulomb interactions, and 2) energy-decreasing caused by a gap in the spin exchange energy. When the modulation value d is small, the increasing value in the classical Coulomb energy is proportional to d^2, and the decreasing value in the spin exchange energy is proportional to $|d|$. Therefore, the total energy for a small modulation is smaller than one in non-modulation case. Therefore, this modulation occurs actually. For ν=2/3, there are two electrons inside one unit-cell as shown in Figure 1. We change the intervals wide in the first unit-cell, and change the intervals narrow in the second unit-cell, and so on. Then, the coupling constant between electrons in nearest orbital pair changes from the original value to two values ξ and ξ', and also the coupling constant between electrons in second nearest orbital pair changes from the original value to two values η and η'. We neglect the difference between η and η', because its contribution to the spin polarization is small. Accordingly, we have the three kinds of coupling constants ξ, ξ' and η which are illustrated in Figure 2.

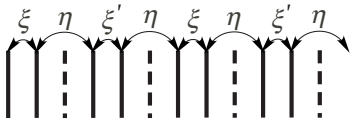

Fig. 2. Interactions with modulation of orbital intervals

Consequently, we obtain a new interaction Hamiltonian as

$$H = \sum_{n=1,2,3,\cdots} [\xi(\sigma^+_{4n-3}\sigma^-_{4n-2} + \sigma^-_{4n-3}\sigma^+_{4n-2}) + \eta(\sigma^+_{4n-2}\sigma^-_{4n-1} + \sigma^-_{4n-2}\sigma^+_{4n-1}) + \xi'(\sigma^+_{4n-1}\sigma^-_{4n} + \sigma^-_{4n-1}\sigma^+_{4n}) + \eta(\sigma^+_{4n}\sigma^-_{4n+1} + \sigma^-_{4n}\sigma^+_{4n+1})] + \sum_{i=1,2,3,\cdots} \frac{\mu_B g}{\mu_0} B \frac{1}{2}\sigma^z_i \quad (2)$$

We can solve the eigenvalue problem of Hamiltonian (2), and we have obtained a new energy gap in the solution. In a real Hall device, there are many random-potentials. Therefore, we should take the effect of these random potentials into consideration. We assume that it is a good approximation to replace the random potential effect with a thermal vibration effect. We therefore introduce an effective temperature T. When we take the values $\frac{\eta}{\xi} = 0.25$, $\frac{\xi'}{\xi} = 1.3$ and $\frac{k_B T}{\xi} = 0.06$, the calculated curve of the spin-polarization has a narrow plateau as shown in Figure 3.

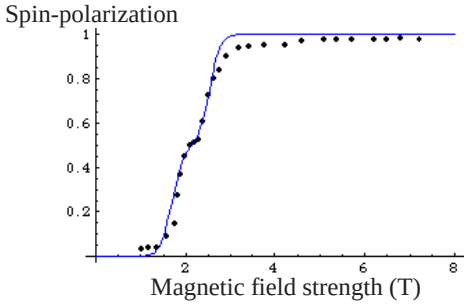

Fig. 3. Experimental data and calculated value of the spin-polarization for the filling factor 2/3

As is seen in this figure, the calculated curve of the spin-polarization is in a good agreement with the experimental data. In the data of Kukushkin et al [1], other polarization curves for seven filling factors also show both wide plateaus and narrow plateaus. These behaviors can be well explained by making use of our method in this paper.

References

[1] I. V. Kukushkin, K. v. Klitzing, and K. Eberl, Phys.Rev.Lett.82 (1999) 3665.

[2] R. B. Laughlin, Phys. Rev. B23 (1981) 5632; B. I. Halperin, Phys. Rev. B25 (1982) 2185

[3] R. Tao and D. J. Thouless, Phys. Rev. B 28 (1983) 1142; R. Tao, Phys. Rev. B 29 (1984) 636.

[4] S. Sasaki, Physica B 281 (2000) 838. S. Sasaki, Proc. 25th Int. Conf. Phys. Semicond., (Springer 2001) 925.

[5] S. Sasaki, Phys. Rev. E, 53 (1996) 168.

[6] R. E. Peierls, Quantum Theory of Solids (Oxford University, London, 1955).