Coulomb Blockade of Phase Slip in Mesoscopic Charge-Density-Wave Systems

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Abstract

Thermally activated phase-slip processes near electrical contacts are responsible for the nonlinear current-voltage characteristic in clean mesoscopic charge-density-wave systems. We calculate the phase-slip rate for a dislocation-loop nucleation and that for a dislocation-pair nucleation, by taking account of the influence of long-range Coulomb interactions. It is shown that due to a large charging energy associated with dislocation lines, the phase-slip rate is greatly reduced for both the nucleation processes when the applied voltage is smaller than a threshold voltage. This indicates that the collective current below the threshold voltage is strongly suppressed by the Coulomb blockade of the phase slip.

Key words: charge density wave; phase slip; Coulomb blockade; mesoscopic system

1. Introduction

The sliding motion of the charge-density wave (CDW) must be accompanied by the conversion of an injected single-particle current into a moving CDW in the vicinity of an electrical contact [1–8]. It has been accepted that the current conversion occurs via thermally activated phase-slip processes. The phase slip is induced by a dislocation-loop (DL) nucleation in usual three-dimensional systems. Ramakrishna *et al.* [2] calculated the phase-slip rate for the DL nucleation at temperature T, and obtained a nonlinear current-voltage characteristic $I_{\mathrm{CDW}} \propto \exp(-\mathrm{const.}/V_{\mathrm{a}}T)$, where V_{a} is a bias voltage applied between a pair of electrical contacts. In contrast, if a sample is very thin, the phase slip due to a dislocation-pair (DP) nucleation becomes more important than the DL nucleation. Maki [4] studied the phase slip due to the DP nucleation and found that $I_{\mathrm{CDW}} \propto V_{\mathrm{a}}^{\alpha}$, where the exponent α depends on T.

We note that the role of long-range Coulomb interactions is completely neglected in both the argument by Ramakrishna *et al.* and that by Maki, so the resulting current-voltage characteristics are justified only when the Coulomb interaction is well screened by quasiparticles. However, quasiparticle screening is far from complete in most CDW materials, such as $K_{0.3}MoO_3$, TaS_3 and $(TaSe_4)_2I$, at low temperatures.

In this paper we study the long-range Coulomb-interaction effect on the thermal nucleation rate of the phase slip. We consider clean samples of mesoscopic dimensions, where the role of impurity pinning is less important. Our attention is focused on the temperature regime where $T \ll T_{\mathrm{c}}$ (T_{c}: transition temperature) but quantum nucleation processes are still negligible. We set $\hbar = k_{\mathrm{B}} = 1$.

2. Phase-slip rate

We first study the phase-slip rate for the DL nucleation [7]. To do so we must evaluate the nucleation energy E_{nuc} of the DL of radius R. If E_{DL} is obtained,

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the phase-slip rate is given by $\Gamma \propto \exp(-\varepsilon_{\rm nuc}/T)$, where $\varepsilon_{\rm nuc} = \max_R\{E_{\rm nuc}\}$. This results in $I_{\rm CDW} \propto \exp(-\varepsilon_{\rm nuc}/T)$. We decompose $E_{\rm nuc}$ as $E_{\rm nuc} = E_{\rm DL} + E_{\rm es}$, where $E_{\rm DL}$ is the excess energy due to the DL and $E_{\rm es}$ is the electrostatic energy caused by the bias voltage $V_{\rm a}$.

We calculate $E_{\rm DL}$ as a function of R. In terms of the phase field θ and the scalar potential φ, the free energy is given by

$$F = \int {\rm d}^3 r \left[\frac{v_{\rm F}}{4\pi s} \left((\partial_x \theta)^2 + \gamma^2 (\nabla_\perp \theta)^2 \right) + \frac{{\rm i}e}{\pi s} \varphi \partial_x \theta + \frac{1}{8\pi} (\nabla \varphi)^2 \right], \tag{1}$$

where s and γ are the unit area of a single chain and the anisotropy constant, respectively. Variation of F leads to

$$\left(\partial_x^2 + \gamma^2 \nabla_\perp^2 \right) \theta(\boldsymbol{r}) + \frac{2}{v_{\rm F}} \partial_x \Phi(\boldsymbol{r}) = 0, \tag{2}$$

$$\nabla^2 \Phi(\boldsymbol{r}) + \frac{v_{\rm F} \lambda_{\rm ph}^{-2}}{2} \partial_x \theta(\boldsymbol{r}) = 0, \tag{3}$$

where $\Phi = {\rm i}e\varphi$ and $\lambda_{\rm ph} = \sqrt{s v_{\rm F}/8e^2}$. Brazovskii and Matveenko [9] solved Eqs. (2) and (3) taking account of the unit vorticity around the DL, and found that the excess energy is

$$E_{\rm DL} = \gamma \left(\frac{\pi}{2} \frac{v_{\rm F} \pi R^2}{s \lambda_{\rm ph}} + c \frac{v_{\rm F} \pi R}{s} \right), \tag{4}$$

where c is a numerical constant of the order of unity. The first term in Eq. (4) represents the charging energy due to the charge imbalance caused by the DL, while the second term comes from the internal loop energy.

Combination of Eq. (4) and the electrostatic energy, which is obtaine as $E_{\rm es} = -eV_{\rm a}\pi R^2/s$ [2], yields

$$E_{\rm nuc} = (eV_{\rm th} - eV_{\rm a}) \frac{\pi R^2}{s} + c\gamma \frac{v_{\rm F} \pi R}{s}, \tag{5}$$

where $V_{\rm th} = (\pi/2)\gamma v_{\rm F}/e\lambda_{\rm ph}$. We see that $E_{\rm nuc}$ monotonically increases with an increase in R when $V_{\rm a}$ is smaller than the threshold voltage $V_{\rm th}$. Thus, unless the transverse size of the sample is extremely small, the phase slip is forbidden when $V_{\rm a} < V_{\rm th}$. Consequently, the collective CDW current cannot flow. This is attributed to the large charging energy associated with the DL nucleation, so we call this phenomenon the Coulomb blockade of the phase slip [7]. Above $V_{\rm th}$, we calculate $\varepsilon_{\rm nuc}$ by minimizing $E_{\rm nuc}$ with respect to R. Finally, we obtain

$$I_{\rm CDW} \propto \exp\left(-\frac{\gamma^2 \pi c^2}{4} \frac{v_{\rm F}^2/s}{e\,(V_{\rm a} - V_{\rm th})\,T} \right), \tag{6}$$

for $V_{\rm a} > V_{\rm th}$. Note that the threshold voltage is rewritten as $V_{\rm th} = (\pi/2)\gamma\omega_{\rm p}/e$ ($\omega_{\rm p}$: plasma frequency in the normal state).

We turn to the phase-slip rate for the DP nucleation. Let L be the thickness of our thin sample. We evaluate the nucleation energy $E_{\rm nuc}$ of the DP separated by a distance R. By adapting Brazovskii and Matveenko's argument to the present problem, we obtain the excess energy for the DP

$$E_{\rm DP} = \gamma \left(\frac{\pi}{2} \frac{v_{\rm F} L R}{s \lambda_{\rm ph}} + c \frac{v_{\rm F} L}{s} \right), \tag{7}$$

where c is a numerical constant of the order of unity. Combining Eq. (7) and the electrostatic energy, $E_{\rm es} = -eV_{\rm a}LR/s$ [4], we find that the nucleation energy $E_{\rm nuc} = E_{\rm DP} + E_{\rm es}$ is

$$E_{\rm nuc} = (eV_{\rm th} - eV_{\rm a}) \frac{LR}{s} + c\gamma \frac{v_{\rm F} L}{s}, \tag{8}$$

where $V_{\rm th}$ has been defined below Eq. (5). Note that Eq. (8) is valid when $R \gg L$. If $R \ll L$, the nucleation energy is given by Eq. (5) instead of Eq. (8). From Eqs. (5) and (8) we see that the Coulomb blockade of the phase slip also arises in this case when $V_{\rm a} < V_{\rm th}$. When $\gamma v_{\rm F}/2L \gg eV_{\rm a} - eV_{\rm th} > 0$, we calculate $\varepsilon_{\rm nuc}$ by minimizing $E_{\rm nuc}$ with respect to R, and finally obtain

$$I_{\rm CDW} \propto \exp\left(-\frac{L^2}{s} \frac{c\gamma v_{\rm F}/L - (eV_{\rm a} - eV_{\rm th})}{T} \right). \tag{9}$$

3. Summary

We find that the CDW current below the threshold voltage $V_{\rm th} \sim \gamma\omega_{\rm p}/e$ is strongly suppressed by the Coulomb blockade of the phase slip caused by a large charging energy associated with dislocation lines. The nonlinear current-voltage characteristics are calculated for both the dislocation-loop nucleation process and the dislocation-pair nucleation process.

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