

# Effects of thermal fluctuations and magnetic field in the SO(5) theory

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## Abstract

Monte Carlo simulations have been performed on O(5) models, which is believed to be able to capture the long-wave length behaviours of the SO(5) theory on the high- $T_c$  superconductivity. In contrast to RG analyses, it is observed that the bicritical fixed point with O(5) symmetry is stable to repulsive biquadratic forces between superconductivity (SC) and antiferromagnetism (AF). An external magnetic field attracts the two orders, and may result in coexistence of AF and SC. A tricritical point is found where phase transitions between disordered and AF states switches between first and second order, while the phase transition into superconducting state (flux-line lattice) is always of first order.

*Key words:* high- $T_c$  superconductivity, SO(5) theory, bicritical point

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## 1. Introduction

The SO(5) theory has been proposed to explain the general temperature vs. doping-rate phase diagrams observed in high- $T_c$  superconductors [1]. An SO(5) symmetry point is expected in the phase diagram as a result of the competition between AF and SC orders. According to the RG theory, however, an O(5) isotropic fixed point becomes unstable in 3D to biquadratic perturbations, which should exist if one takes the asymmetry between the spin and charge gaps in cuprates into account in the SO(5) theory [2,3].

As the RG analyses are based on weak-coupling treatments, and on  $\epsilon$  expansions from four dimensions, the accuracy of its predictions on real materials of three dimensions is sometimes questionable. In order to clarify the situation, we have performed, as a strong coupling treatment, Monte Carlo simulations on the following hamiltonian on simple cubic lattice [4]:

$$H = -J \sum_{\langle i,j \rangle} (\mathbf{s}_i \cdot \mathbf{s}_j + \mathbf{t}_i \cdot \mathbf{t}_j) + g \sum_i \mathbf{s}_i^2 + w \sum_i \mathbf{s}_i^2 \mathbf{t}_i^2,$$

where  $\mathbf{s}$  ( $\mathbf{t}$ ) has 3 (2) components, and  $\mathbf{s}_i^2 + \mathbf{t}_i^2 = 1$ .

## 2. Bicritical O(5) symmetric point

Since it has been revealed that quantum fluctuations originated from the Gutzwiller projection out of double occupancy of electrons produce a positive  $w$  field [3], we concentrate on repulsive AF-SC cases. Without losing generality, we present here data for  $w = 0.1J$ . As shown in Fig. 1, for  $g \geq 0.012J$  we always observe SC long-range order at low temperatures. The disorder to SC phase transitions are clearly continuous, namely of second order. On the other side, for  $g \leq 0.010J$  the long-range order is AF, as shown in Fig. 2, and the transitions are second order. At  $g = 0.011J$ , we observe either AF or SC depending on the initial condition and/or annealing process. Therefore, it is safe to say that there is no co-existence between AF and SC in a  $T - g$  phase diagram. From the above observations, we arrive at the conclusion that a bicritical point locates at  $g = 0.011J$ . We have performed simulations on other values of  $w$  field up to  $0.5J$ . The bicritical point is robust as far as  $w$  is non negative [5].

According to RG analyses, the stable fixed point should be a decoupled, tetracritical one located in the parameter space  $w < 0$  [6]. For positive  $w$  fields, RG predicts fluctuation-induced first-order disorder to SC

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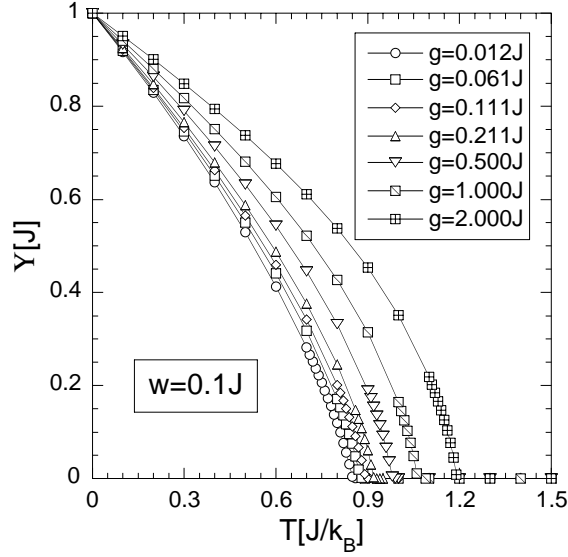


Fig. 1. The helicity modulus (which is proportional to the superfluid density) for the  $g$  fields larger than the bicritical value  $g_b = 0.011J$ .

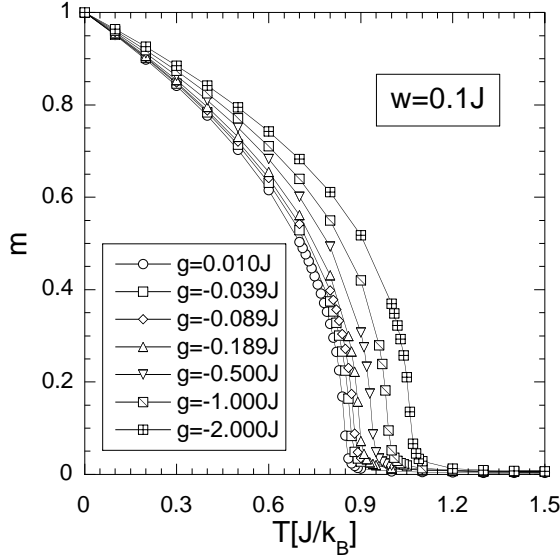


Fig. 2. The Néel order parameter for the  $g$  fields less than the bicritical value  $g_b = 0.011J$ .

and/or AF transition. Therefore, our simulation results contradict with the picture derived by RG. While the data shown in Figs. 1 and 2 are for  $L^3 = 40^3$ , recently we have simulated up to systems of  $L^3 = 80^3$  and observed there is almost no size effect [5]. Therefore, although the existence of a very tiny regime around the point disorder, AF and SC phases meeting where disorder to AF and/or SC transitions become first-order cannot be excluded completely, we conclude that the  $O(5)$  symmetric point governs almost all the phase di-

agram, and leaves significant effects in various physical properties.

We have successfully developed a scaling theory which describes the crossover phenomena observed in Figs. 1 and 2 [4]. Employing this scaling theory, we estimated the critical exponents  $\nu_5 \simeq 0.728 \pm 0.018$ ,  $\beta_5 \simeq 0.400 \pm 0.002$ , and crossover exponent  $\phi \simeq 1.387 \pm 0.030$ , and the bicritical point  $T_b \simeq 0.8458 \pm 0.0005J/k_B$ .

### 3. Effects of magnetic field

When a magnetic field is applied the Néel order parameter responds to it by forming the so-called spin flopped state. On the other hand, the magnetic field also induces orbital effects, and the superconductivity order parameter feels gauge modulation. This results in quantized vortices in the system. The phase transition from disordered state to superconducting state is accompanied by the melting of the flux line lattice, and is therefore first order [8].

In the present system, AF components are enhanced at the vortex cores, where SC order parameter is suppressed. Therefore, the magnetic field induces effectively attraction between AF and SC. In our simulations, we find that there appears a regime in the  $T - g$  phase diagram where long-range orders of AF and SC (in the fashion of flux line lattice) coexist, even for positive  $w$  fields. Furthermore, the phase transition associated with long-range AF order becomes first order near the coexistence regime. There is a tricritical point on the AF phase boundary from which the disorder to AF transition becomes second order when the SC fluctuations are small [9].

Simulations have been performed on the Numerical Materials Simulator (SX-5) in NIMS, Japan.

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